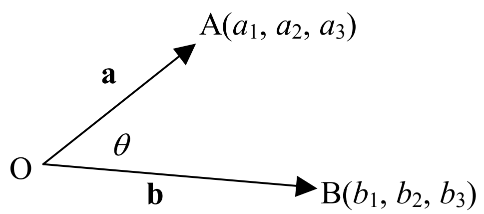
C3.Q101.NOTES

Vector Basics: Dot Product Summary (12.3)



$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, θ is the angle between **a** and **b** $(0 \le \theta \le 2\pi)$

THM: $a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$ [Prove using the Law of Cosines]

DEFN: $a_1b_1 + a_2b_2 + a_3b_3 = \mathbf{a} \cdot \mathbf{b}$

THM (after substitution): $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

COR: $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

THM: Vectors **a** and **b** are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$

The work done by a constant force **a** as its point of application moves along the vector **b** is $\mathbf{a} \cdot \mathbf{b}$
$W = \mathbf{a} \cdot \mathbf{b}$

<u>Scalar projection of **b** onto **a**</u>:

The magnitude of the force from **b** being applied along **a** is $comp_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|}$

<u>Vector projection of **b** onto **a**</u>:

The vector representation of the force from **b** being applied along **a** is $proj_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$

Vector Basics: Cross Product Summary (12.4)

DEFN: Vector (Cross) Product: $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

THM: The vector $\mathbf{a} \times \mathbf{b}$ is orthogonal to both **a** and **b**.

Geometric Application: The vector $\mathbf{a} \times \mathbf{b}$ is normal to the plane containing both **a** and **b**.

THM: $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$

Geometric Application: $|\mathbf{a} \times \mathbf{b}|$, the magnitude of vector $\mathbf{a} \times \mathbf{b}$, is the area of the parallelogram determined by **a** and **b**.

THM: Vectors **a** and **b** are parallel if and only if $\mathbf{a} \times \mathbf{b} = 0$

LESSON 1 (12.6)

I. **Quadratic Surfaces**

NAME OF SHAPE:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

Trace	Equation of trace	Description of trace	Sketch of trace

NAME OF SHAPE:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Trace	Equation of trace	Description of trace	Sketch of trace

NAME OF SHAPE:

$$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

Trace	Equation of trace	Description of trace	Sketch of trace

NAME OF SHAPE:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

Trace	Equation of trace	Description of trace	Sketch of trace

Example1: Sketch and identify $16x^2 - 9y^2 + 36z^2 = 144$

Example2: Sketch and identify $y^2 + 4z^2 = x$.

Example3: Sketch and identify $z = 2 - 3x^2 - y^2$

MATCHING:

Exer. 9 – 20: Match each graph with one of the equations.

A. $\dfrac{x^2}{25}+\dfrac{y^2}{9}+\dfrac{z^2}{4}=1$

B. $x=z^2+\dfrac{y^2}{4}$

C. $y^2+z^2-x^2=1$

D. $\dfrac{x^2}{4}+\dfrac{y^2}{9}-\dfrac{z^2}{4}=0$

E. $z=\dfrac{x^2}{9}-\dfrac{y^2}{4}$

F. $z^2-\dfrac{x^2}{4}-y^2=1$

G. $\dfrac{z^2}{9}+\dfrac{y^2}{4}-\dfrac{x^2}{4}=0$

H. $\dfrac{x^2}{4}-y^2-z^2=1$

I. $y=\dfrac{x^2}{4}-\dfrac{z^2}{9}$

J. $x^2+\dfrac{y^2}{4}+\dfrac{z^2}{16}=1$

K. $z=\dfrac{x^2}{9}+y^2$

L. $\dfrac{x^2}{4}+\dfrac{y^2}{16}+\dfrac{z^2}{9}=1$

M. $y=\dfrac{z^2}{9}-\dfrac{x^2}{4}$

N. $y=\dfrac{x^2}{4}+\dfrac{z^2}{4}$

O. $z^2+\dfrac{x^2}{4}-y^2=1$

P. $\dfrac{x^2}{4}+\dfrac{z^2}{9}-\dfrac{y^2}{4}=0$

Q. $y^2-\dfrac{x^2}{4}-z^2=1$

R. $x^2+\dfrac{y^2}{4}-z^2=1$

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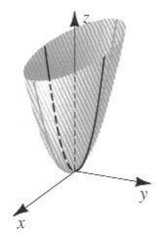

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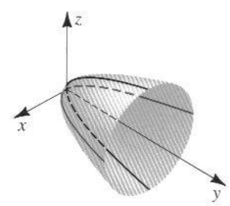

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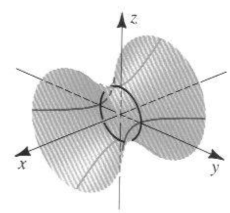

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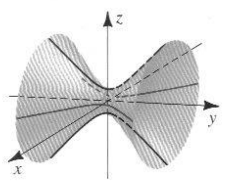

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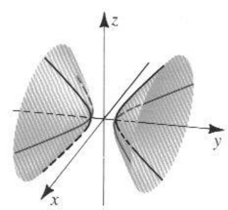

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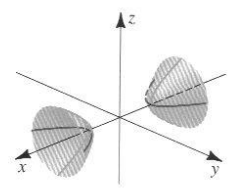

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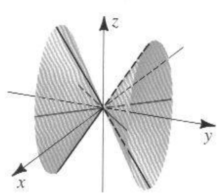

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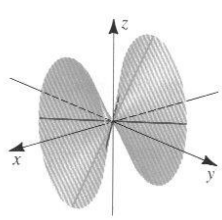

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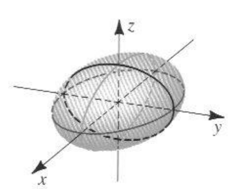

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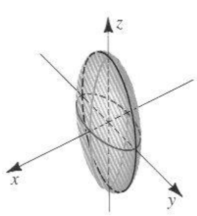

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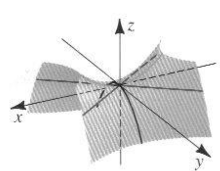

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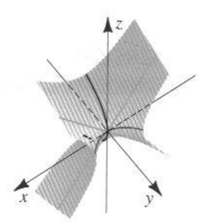

II. **Cylinders**

Definition of **Cylinder:**

Sketch each graph in $\Re^2$ and $\Re^3$.

1. $\frac{x^2}{4}+\frac{y^2}{9}=1$

2. $y^2 = 9 - z$

3. $z = \sin x$

4. $y = 3$

LESSON 2 (15.7, 15.8)

I. Find an equation in x and y whose graph contains the points on the curve C. Include domain and range.

1. $x = t - 2, \quad y = 2t + 3; \quad 0 \le t \le 5$
2. $x = e^t, \quad y = e^{-2t}; \quad t \in \Re$
3. $x = 2\sin t, \quad y = 3\cos t; \quad 0 \le t \le 2\pi$
4. $x = \cosh t, \quad y = \sinh t; \quad t \in \Re$

II. Find an equation in x and y that has the same graph as the polar equation.

1. $r \sin \theta = 0$
2. $r = 4 \csc \theta$
3. $r \cos\theta + r \sin \theta = 1$
4. $r^2 = 4r \sin \theta$
5. $r^2 \sin 2\theta = 2$
6. $r = a \sin \theta$
7. $r = \cot\theta \csc \theta$

III. Find a polar equation that has the same graph as the equation in x and y.

1. $y = x$
2. $x = 7$
3. $x^2 + y^2 = 4$
4. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$
5. $x^2 + (y - 2)^2 = 4$

IV. Parametric, Polar, Cylindrical and Spherical

1. A) Find the points on the curve C: $x = 4t^2, \quad y = t^3 - 12t; \quad t \in \Re$: at which the tangent line is either horizontal or vertical. B) Find $\dfrac{d^2 y}{dx^2}$.
2. Find the length of the curve C: $x = e^t \cos t, \quad y = e^t \sin t; \quad 0 \le t \le \pi / 2$.
3. Find the slope of the tangent line to the graph of $r = -2 \sin \theta$ at $\theta = \dfrac{\pi}{6}$.
4. Change the cylindrical coordinates $(1, \pi, e)$ and $(1, 3\pi / 2, 5)$ into rectangular coordinates.
5. Change the rectangular coordinates $(2\sqrt{3}, 2, -1)$ and $(4, -3, 2)$ into cylindrical coordinates.
6. Change the spherical coordinates $(5, \pi, \pi / 2)$ and $(4, 3\pi / 4, \pi / 3)$ into rectangular coordinates.
7. Change the rectangular coordinates $(0, \sqrt{3}, 1)$ and $(-1, 1, \sqrt{6})$ into spherical coordinates.

Relationship between Rectangular and Polar Coordinates:

Relationship between Rectangular and Cylindrical Coordinates:

Relationship between Rectangular and Spherical Coordinates:

You will need extra notebook paper here.

LESSON 3 (12.5)

Vector Equation for a Line:

Vector Equation for a Line Segment:

Vector Equation of a Plane: