## C3.Q104.NOTES: 15A DOUBLE INTEGRALS

# LESSON 1 (15.1 – 15.3)

WARM UP: Find the area of the region *R* bounded by  $y = x^2$  and y = 2x



1. Describe some possible meanings of the double integral:  $\iint_{R} f(x, y) dA$ 

2. Express 
$$\iint_{R} f(x, y) dA$$
 as  $\iint_{R} f(x, y) dy dx$  and  $\iint_{R} f(x, y) dx dy$ 

3. Evaluate 
$$\iint_{R} f(x, y) dA$$
 for  $f(x, y) = 1$ 

4. Evaluate  $\iint_{R} f(x, y) dA$  for  $f(x, y) = x^{3} + 4y$ 

5. Express 
$$\iint_{R} f(x, y) dA$$
 as  $\iint_{R} f(x, y) dy dx$  and  $\iint_{R} f(x, y) dx dy$  for

R = Region bounded by:  $x = y^3$ , x + y = 2, y = 0

6. Express 
$$\iint_{R} f(x, y) dA$$
 as  $\iint_{R} f(x, y) dy dx$  OR  $\iint_{R} f(x, y) dx dy$ 

R = Region bounded by:  $2y = 16 - x^2$ , x + 2y = 4

7. Draw the region R in 
$$\int_{1}^{3} \int_{\pi/6}^{y^2} f(x, y) dx dy$$
. Evaluate 
$$\int_{1}^{3} \int_{\pi/6}^{y^2} f(x, y) dx dy$$
 for  $f(x, y) = 2y \cos(x)$ 

8. Evaluate  $\iint_{R} f(x, y) dA$  where R is the rectangular region  $[1,4] \times [-1,2]$  and  $f(x, y) = 2x + 6x^2 y$ 

### LESSON 1 (15.1 – 15.3) HOMEWORK

Sketch the region bounded by the graphs of the given equations.

Express 
$$\iint_{R} f(x, y) dA$$
 as  $\iint_{R} f(x, y) dy dx$  and  $\iint_{R} f(x, y) dx dy$   
1.  $y = \sqrt{x}$   $x = 4$   $y = 0$   
2.  $y = \sqrt{x}$   $x = 0$   $y = 2$   
3.  $y = \sqrt{x}$   $y = x^{3}$   
4.  $8y = x^{3}$   $y - x = 4$   $4x + y = 9$ 

5. Find the area bounded by y = x y = 3x x + y = 4

6. Find the volume of a lake whose surface is define by the region R in #5 and whose depth is defined by f(x, y) = x + y

7. Find the volume of the solid whose base is the region R bounded by  $y^2 = -x$ , x - y = 4, y = -1and y = 2 and whose height is f(x, y) = xy.

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## LESSON 2 (15.1 – 15.3)

WORKSHOP: Section 15.3: #5, 7, 17, 21, 43, 45, 51, 55; Section 15.1 #1, 9; Section 15.2, #9,17

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LESSON 3 (15.4, 15.9)

1. (WARM UP) SET UP only the integral used to find the volume of the solid bounded by  $z = 4 - x^2 - y^2$  and the *xy*-plane.

2. Find the volume of the solid described in #1 using a polar transformation.

3. Evaluate  $\iint_{R_{xy}} e^{-(x^2+y^2)} dx dy$  for the region *R* bounded in the first quadrant by the circles  $x^2 + y^2 = 1$  $x^2 + y^2 = 4$ .

4. Show that the transformation  $T: x = r \cos \theta$ ,  $y = r \sin \theta$  always yields |J| = r.

5. Evaluate  $\iint_{R_{xy}} e^{(y-x)/(y+x)} dx dy$  where R is the region within the trapezoid defined by the points (0,1)(0,2)(2,0)(1,0).