CALCULUS 3

Q102

PARTIAL DERIVATIVES
GRADIENTS

DIRECTIONAL DERIVATIVES

C3 Q102 LESSON 1

PART I: MULTIVARIABLE FUNCTIONS: $f:\mathbb{R}^2 \mapsto \mathbb{R}^1$

(INTRODUCTION AND ILLUSTRATIONS)

PART II: FIRST PARTIAL DERIVATIVES

A. First Partial Derivatives at a Point

1. Let $z = f(x, y)$. Assume f is continuous and differentiable.

The table below gives values of f at certain (x, y) coordinates.

		x					
		0	1	2	3	4	5
	0	2.0	3.0	4.0	5.0	6.0	7.0
	1	4.0	5.5	6.0	5.75	5.0	4.0
	2	6.0	6.0	6.0	6.0	6.0	6.0
y	3	10.0	7.0	5.0	6.5	5.1	6.9
	4	12.0	8.5	4.5	7.5	5.0	7.0
	5	18.0	9.0	3.0	8.5	4.5	8.0

Suppose $f(x, y)$ represents the temperature (^{o}F) at points (x, y) where x and y are measured in inches.

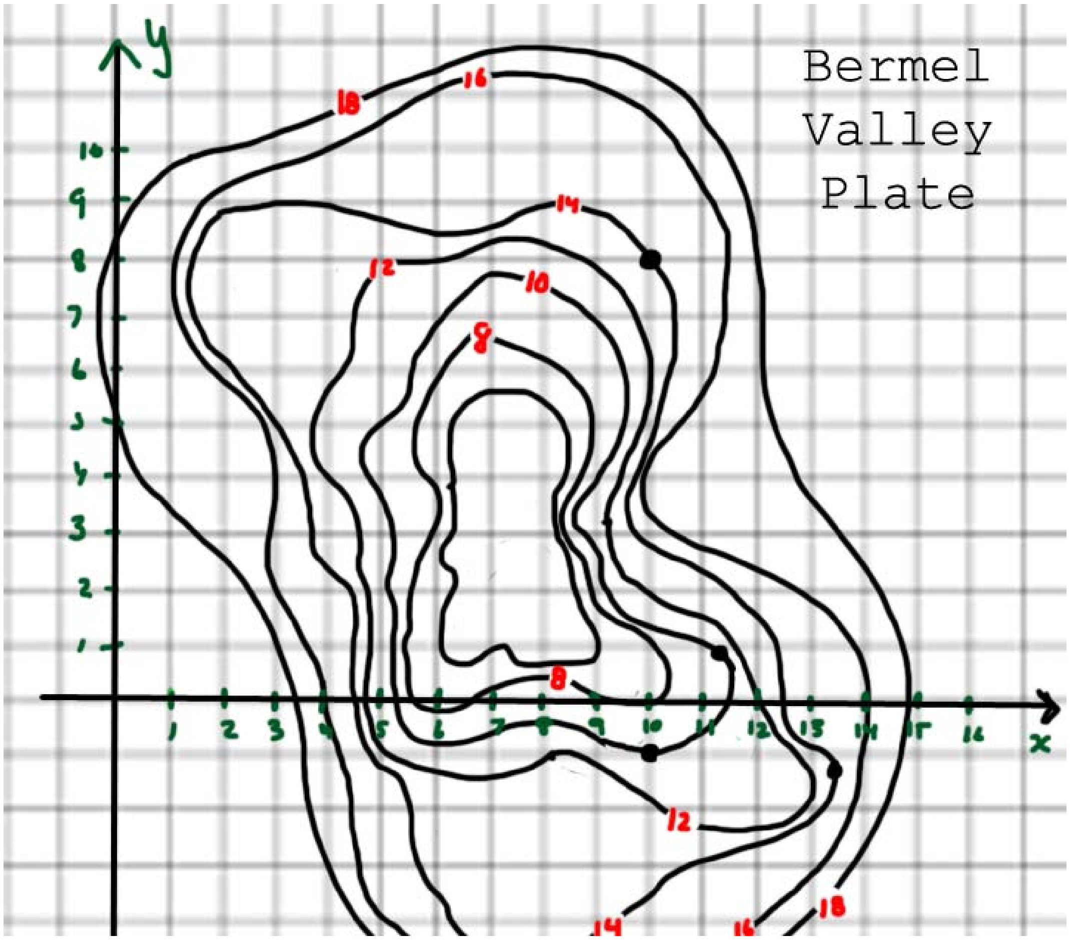
OR

Suppose $f(x, y)$ represents the altitude of a range (miles) at points (x, y) where x and y are measured in km.

Estimate and interpret: $f_x(1,4)$, $f_y(1,4)$, $f_x(4,2)$, $f_y(4,2)$.

2. Let $z = f(x, y)$. Assume f is continuous and differentiable.

The level curve map for f is given below.



Suppose $f(x, y)$ represents the temperature (°F) at points (x, y) where x and y are measured in inches.

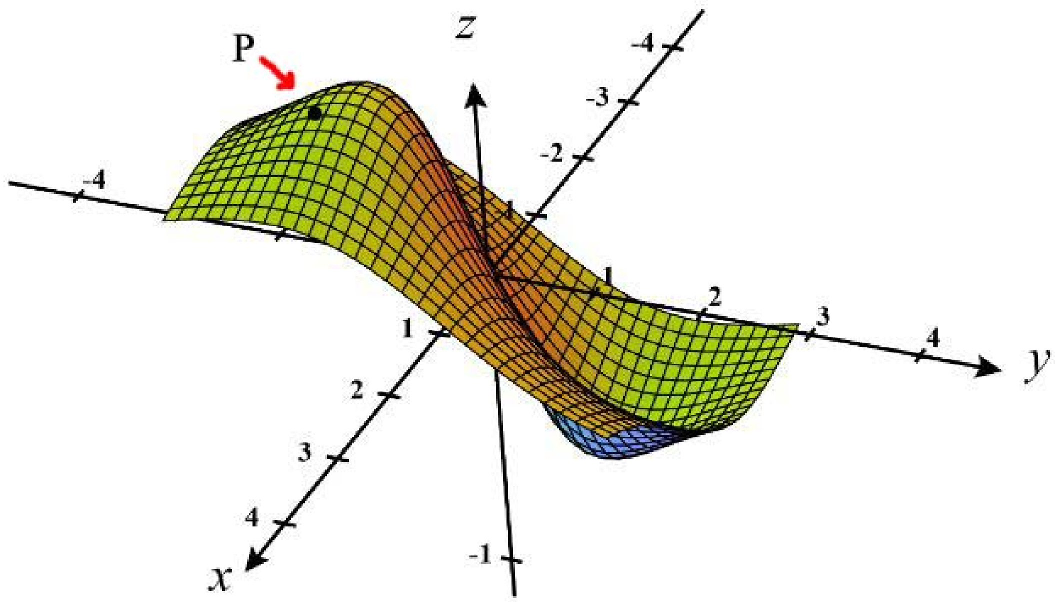
OR

Suppose $f(x, y)$ represents the altitude of a range (miles) at points (x, y) where x and y are measured in m.

Estimate and interpret: $f_x(10,-1)$, $f_y(10,-1)$

3. Let $z = f(x, y)$. Assume f is continuous and differentiable.

The graph of f is shown below.



Suppose $f(x, y)$ represents the temperature (°F) at points (x, y) where x and y are measured in inches.

OR

Suppose $f(x, y)$ represents the altitude of a range (miles) at points (x, y) where x and y are measured in m.

Determine if each of the following are a positive or negative value.

f_x at point P.

f_y at point P.

B. First Partial Derivative Functions

Definition: Let f be a function of x and y.

The **first partial derivative of *f* with respect to *x*** is the function

$$f_x(x, y) = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$ provided it exists.

The **first partial derivative of *f* with respect to *y*** is the function

$$f_y(x, y) = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$ provided it exists.

First Partial Derivative Techniques

Example1: $f(x, y) = x^3 y^2 - 2x^2 y + 3x$

Find the first partial derivatives.

Example2: $z = xy^2 e^{xy}$

Find the first partial derivatives.

C3 Q102 LESSON 2

PART I: GRADIENT AND DIRECTIONAL DERIVATIVE $f : \mathbb{R}^2 \mapsto \mathbb{R}^1$

<u>GRADIENT</u> $f : \mathbb{R}^2 \mapsto \mathbb{R}^1$

<u>DIRECTIONAL DERIVATIVE</u> $f : \mathbb{R}^2 \mapsto \mathbb{R}^1$

(*more on directional derivative*)

Example 1: Let $f(x,y) = x^3 y^2$

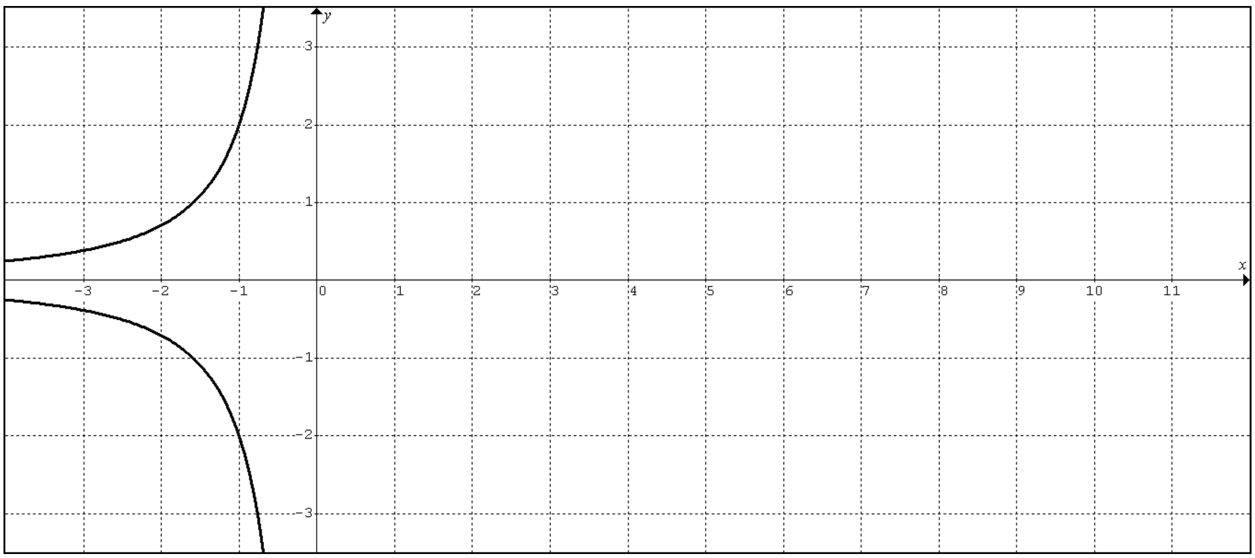
(a) Define the level curve shown below which passes through the point (-1, 2).

(b) In what direction from the point (-1, 2) will *f* have a maximum rate of change?

(c) What is the rate of change in *f* from (-1, 2) in the direction of $\mathbf{v} = \langle 4, -3 \rangle$?

(d) If *f*(*x*,*y*) is temperature (^{o}F) at (x, y) and *x* and *y* are in meters, provided an interpretation of the answer found in part (c).

(e) Graphically illustrate parts (b) and (c).

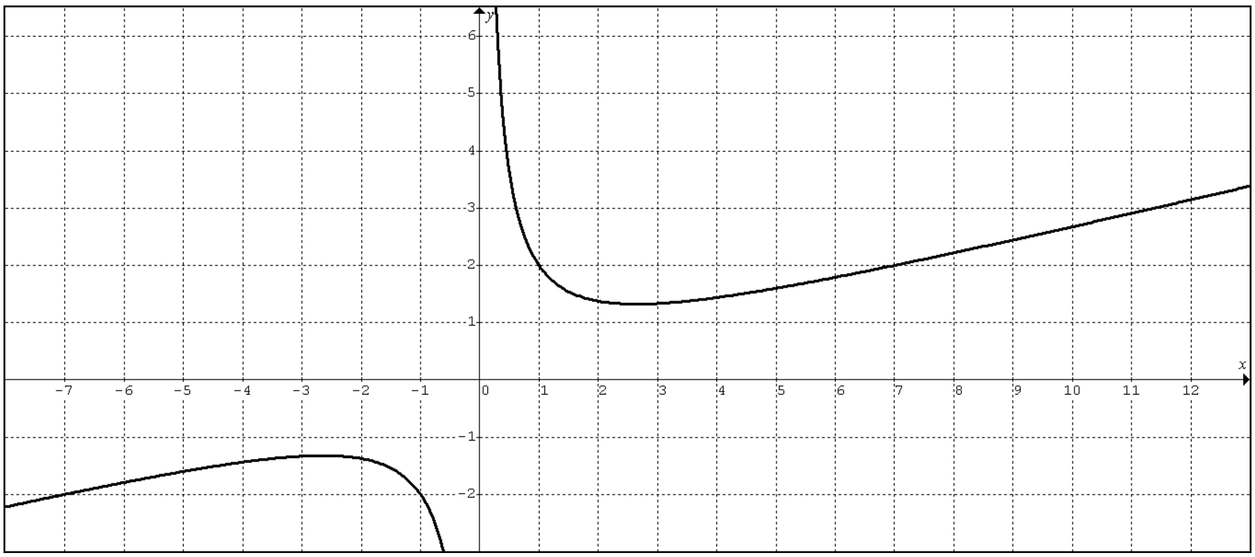


Example 2: Let $f(x, y) = x^2 - 4xy$

(a) Define the level curve shown below which passes through the point (1, 2)

(b) Find the directional derivative of *f* at P(1,2) in the direction from P(1,2) to Q(2,5).

(c) Illustrate part (b) graphically.



Example 3: Let $f(x, y) = 2 + x^2 + \frac{1}{4}y^2$

(a) Define the level curve of f that passes through the point P(1,2)

(b) Find the direction in which *f*(*x*,*y*) increases most rapidly at the point P(1,2).
Find the direction in which *f*(*x*,*y*) decreases most rapidly at the point P(1,2).

(c) Find the maximum rate of change of *f* at point P.
Find the minimum rate of change of *f* at point P.

PART II: THE CHAIN RULE

(chain rule continued)

IMPLICIT DIFFERENTIATION AND PARTIAL DERIVATIVES

C3 Q102 LESSON 3

FUNCTIONS IN R4 (GENERAL OVERVIEW)

LEVEL SURFACE (GENERAL NOTES)

GRADIENT AND DIRECTIONAL DERIVATIVE

EXAMPLE: $w = x^2 - y^2 + z^2$ **:** $w = f(x, y, z)$

(1) Describe the Shape of the Graph of $w = x^2 - y^2 + z^2$

(2) Define the Domain of $w = x^2 - y^2 + z^2$

Let w = the temperature (^{o}F) at point (x, y, z).

(3) Define the level surfaces at w = 0, 4, -4.

$w = f(x, y, z) = x^2 - y^2 + z^2$

(4) Compute the Gradient $\nabla f(x, y, z)$

(5) Define the level surface that passes through the point (-1, 1, -2).

(6) In what Direction from point (-1, 1, -2) will *w* increase must rapidly? What is the maximum rate of increase leaving point (-1, 1, -2)?

(7) Find the Directional Derivative when moving from point P(-1, 1, -2) to point Q(4,2,-3).

TANGENT PLANE

EXAMPLE: Find an equation of the plane tangent to the surface $x^2 + 3y^2 - z^2 = 0$ at the point (1, 1, 2) on the surface.

C3 Q102 LESSON 4

PART I: SECOND PARTIAL DERIVATIVES

A. Notation and Clairaut's Theorem

Clairaut's Theorem: Let *f* be a function of two variables *x* and *y*. If f, f_x, f_y, f_{xy}, f_{yx} are continuous on an open interval region *R*, then $f_{xy} = f_{yx}$ throughout *R*.

B. MEANING OF SECOND PARTIALS

EX: $f(x,y) = x^3y^2 - 2x^2y + 3x$ Find all second partial derivatives.

PART II: TANGENT PLANE LINEARIZATION