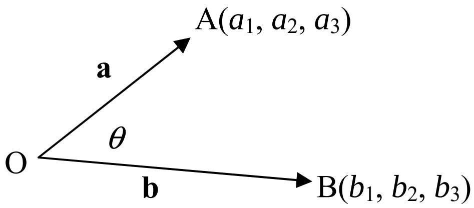
CALCULUS 3
Q101

(3D-SPACE)
$\Re^3$

Vector Basics: Dot Product Summary (12.3)



$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, θ is the angle between **a** and **b** $(0 \le \theta \le 2\pi)$

THM: $a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$ [Prove using the Law of Cosines]

DEFN: $a_1b_1 + a_2b_2 + a_3b_3 = \mathbf{a} \cdot \mathbf{b}$

THM (after substitution): $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

COR: $\cos\theta = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

THM: Vectors **a** and **b** are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b} = 0$

The work done by a constant force **a** as its point of application moves along the vector **b** is $\mathbf{a} \cdot \mathbf{b}$
$W = \mathbf{a} \cdot \mathbf{b}$

<u>Scalar projection of **b** onto **a**</u>:

The magnitude of the force from **b** being applied along **a** is $comp_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|}$

<u>Vector projection of **b** onto **a**</u>:

The vector representation of the force from **b** being applied along **a** is $proj_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{b} \cdot \mathbf{a}}{|\mathbf{a}|^2}\mathbf{a}$

Vector Basics: Cross Product Summary (12.4)

DEFN: Vector (Cross) Product: $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

THM: The vector $\mathbf{a}\times\mathbf{b}$ is orthogonal to both **a** and **b**.

Geometric Application: The vector $\mathbf{a}\times\mathbf{b}$ is normal to the plane containing both **a** and **b**.

THM: $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$

Geometric Application: $|\mathbf{a}\times\mathbf{b}|$, the magnitude of vector $\mathbf{a}\times\mathbf{b}$, is the area of the parallelogram determined by **a** and **b**.

THM: Vectors **a** and **b** are parallel if and only if $\mathbf{a}\times\mathbf{b} = 0$

R^2 SPACE – Rectangular and Polar Coordinate Relationships

R^2 SPACE – Quadratic Relationships

$\Re^3$ — SPACE

I. Points

A. Rectangular Coordinates
B. Cylindrical Coordinates
C. Spherical Coordinates

II. Lines

A. Parametric Equations
B. Vector Valued Functions $f : U \subset R^1 \mapsto R^3$

III. Surface Equations and Graphs

A. Planes
B. Quadratic Surfaces
C. Quadratic Functions $f : U \subset R^2 \mapsto R^1$
D. Cylinders

LESSON 1 – POINTS IN R^3 (15.7, 15.8)

€ Relationship between Rectangular and Cylindrical Coordinates:

€ Relationship between Rectangular and Spherical Coordinates:

LESSON 1 – EXAMPLES

1. Change the cylindrical coordinates $(1, \pi, e)$ and $(1, 3\pi / 2, 5)$ into rectangular coordinates.

2. Change the rectangular coordinates $\left(2\sqrt{3}, 2, -1\right)$ and $\left(4, -3, 2\right)$ into cylindrical coordinates.

3. Change the spherical coordinates $(5, \pi/2, \pi)$ and $(4, \pi/3, 3\pi/4)$ into rectangular coordinates.

4. Change the rectangular coordinates A. $\left(0,\sqrt{3},1\right)$ B. $\left(-1,1,\sqrt{6}\right)$ C. $\left(1,-1,\sqrt{6}\right)$ D. $\left(-1,-1,-\sqrt{6}\right)$ into spherical coordinates.

LESSON 2 (12.5)

Parametric and Vector Equations for a Line

LESSON 2 (12.5)

Equation of a Plane

EX1: Write a vector normal to the plane $2x - 3y + z = 6$.

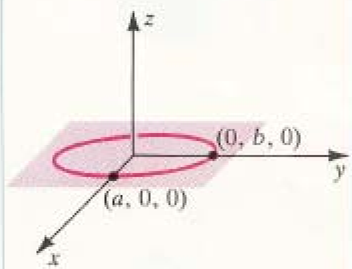
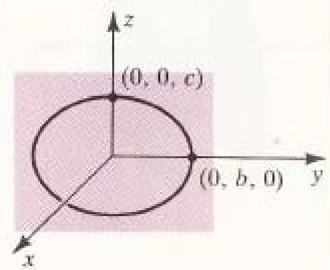
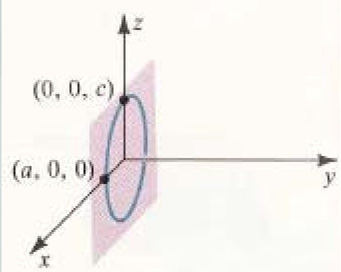
EX2: Write the equation of the plane through P(1, 2, 3), Q(-2, 1, 0) and R(5, 1, 2).

LESSON 3 (12.6)

Quadratic Surfaces

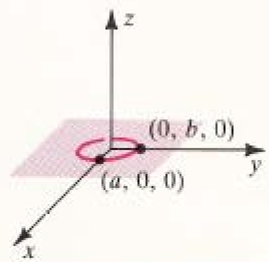
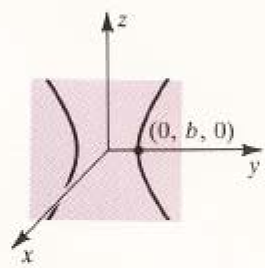
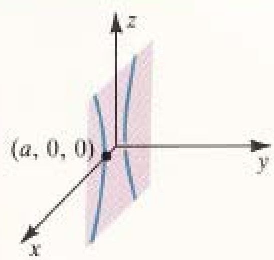
1. $$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

NAME OF SHAPE:

Trace	Equation of trace	Description of trace	Sketch of trace
xy-trace	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	Ellipse	
yz-trace	$\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Ellipse	
xz-trace	$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$	Ellipse	

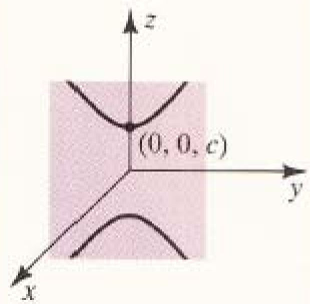
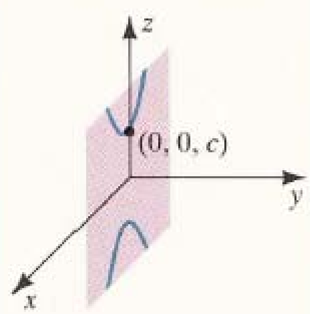
2. $$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

NAME OF SHAPE:

Trace	Equation of trace	Description of trace	Sketch of trace
xy-trace	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	Ellipse	
yz-trace	$\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	Hyperbola	
xz-trace	$\frac{x^2}{a^2}-\frac{z^2}{c^2}=1$	Hyperbola	

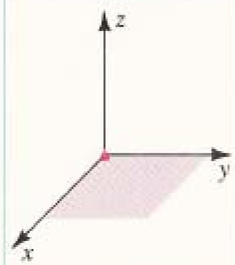
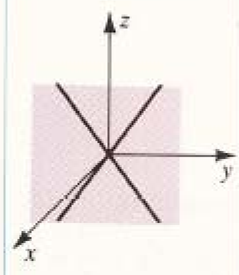
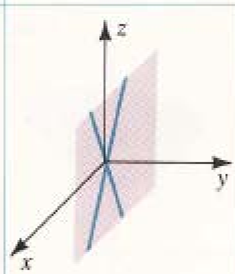
3. $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

NAME OF SHAPE:

Trace	Equation of trace	Description of trace	Sketch of trace
xy-trace	$-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	None	No graph
yz-trace	$-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Hyperbola	
xz-trace	$-\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$	Hyperbola	

4. $$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

NAME OF SHAPE:

Trace	Equation of trace	Description of trace	Sketch of trace
xy-trace	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=0$	Origin	
yz-trace	$\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$	Two intersecting lines	
xz-trace	$\frac{x^2}{a^2}-\frac{z^2}{c^2}=0$	Two intersecting lines	

5. $cz = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $c > 0$

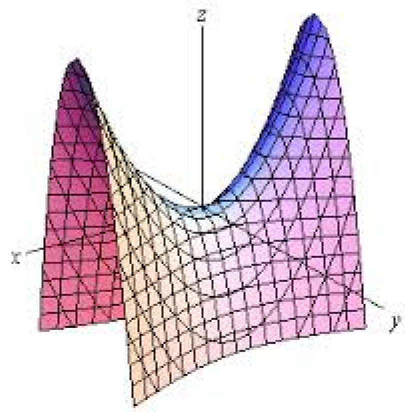
NAME OF SHAPE:

Trace	Equation of trace	Description of trace	Sketch of trace

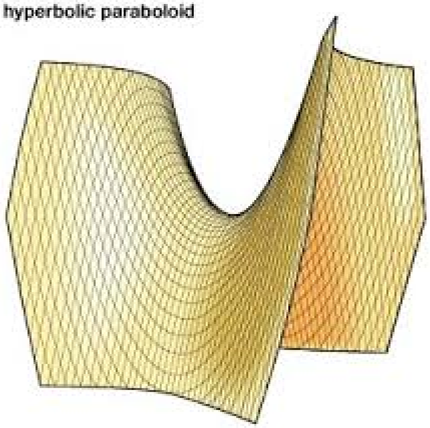
6. $cz = \frac{y^2}{a^2} - \frac{x^2}{b^2}$ $c > 0$

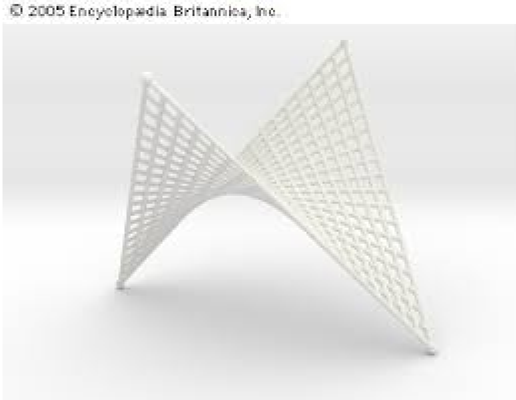
NAME OF SHAPE:

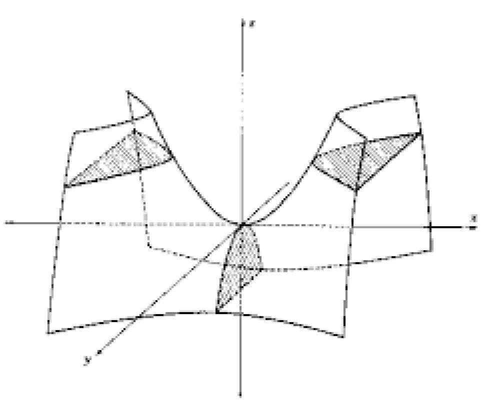
Trace	Equation of trace	Description of trace	Sketch of trace



hyperbolic paraboloid







MATCHING:

Exer. 9 – 20: Match each graph with one of the equations.

A. $\dfrac{x^2}{25} + \dfrac{y^2}{9} + \dfrac{z^2}{4} = 1$

B. $x = z^2 + \dfrac{y^2}{4}$

C. $y^2 + z^2 - x^2 = 1$

D. $\dfrac{x^2}{4} + \dfrac{y^2}{9} - \dfrac{z^2}{4} = 0$

E. $z = \dfrac{x^2}{9} - \dfrac{y^2}{4}$

F. $z^2 - \dfrac{x^2}{4} - y^2 = 1$

G. $\dfrac{z^2}{9} + \dfrac{y^2}{4} - \dfrac{x^2}{4} = 0$

H. $\dfrac{x^2}{4} - y^2 - z^2 = 1$

I. $y = \dfrac{x^2}{4} - \dfrac{z^2}{9}$

J. $x^2 + \dfrac{y^2}{4} + \dfrac{z^2}{16} = 1$

K. $z = \dfrac{x^2}{9} + y^2$

L. $\dfrac{x^2}{4} + \dfrac{y^2}{16} + \dfrac{z^2}{9} = 1$

M. $y = \dfrac{z^2}{9} - \dfrac{x^2}{4}$

N. $y = \dfrac{x^2}{4} + \dfrac{z^2}{4}$

O. $z^2 + \dfrac{x^2}{4} - y^2 = 1$

P. $\dfrac{x^2}{4} + \dfrac{z^2}{9} - \dfrac{y^2}{4} = 0$

Q. $y^2 - \dfrac{x^2}{4} - z^2 = 1$

R. $x^2 + \dfrac{y^2}{4} - z^2 = 1$

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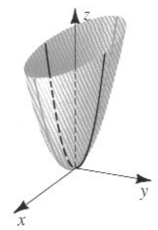

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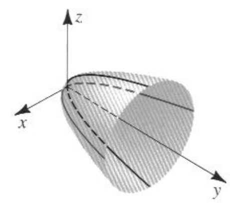

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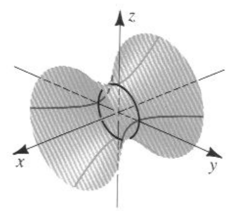

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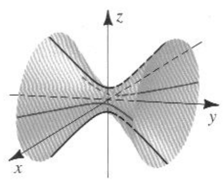

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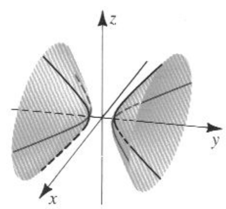

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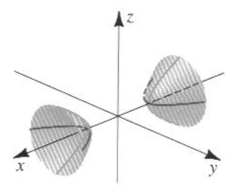

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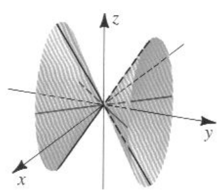

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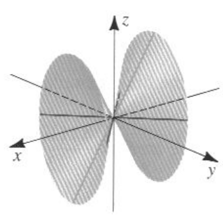

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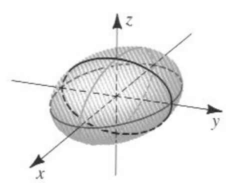

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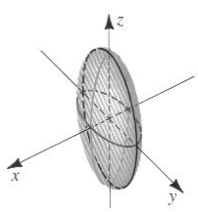

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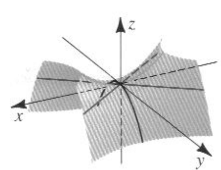

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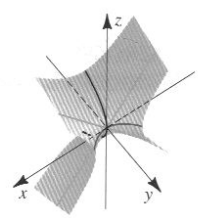

Example1: Sketch, identify, and describe $16x^2 - 9y^2 + 36z^2 = 144$

Example2: Sketch, identify, and describe $y^2 + 4z^2 = x$.

Example3: Sketch, identify, and describe $z = 2 - 3x^2 - y^2$

LESSON 3 (12.6)
Cylinders

Definition of **Cylinder:**

Sketch each graph in $\Re^2$ and $\Re^3$. Identify and describe the surface in $\Re^3$.

1. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

2. $y^2 = 9 - z$

3. $z = \sin x$

4. $y = 3$