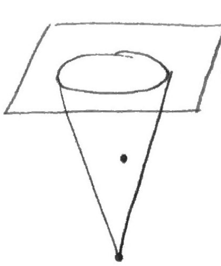
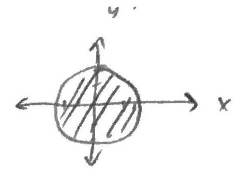
1



$f(x,y,z) = k(x^2+y^2+z^2)$

$2 = \sqrt{x^2+y^2}$

$x^2+y^2 = 4$



A. T: $x = r\cos\theta \quad y = r\sin\theta \quad z = z \quad |J| = r$

$$M = k\int_0^{2\pi}\int_0^{2}\int_r^{2} (r^2+z^2)\, r \, dz\, dr\, d\theta$$

$$= k\int_0^{2\pi}\int_0^{2} \left[r^2 z + \frac{z^3}{3}\right]_r^2 dr\, d\theta = k\int\int\left[\left(2r^2+\frac{8}{3}\right)-\left(r^3+\frac{r^3}{3}\right)\right] r\, dr\, d\theta$$

$$= k\int_0^{2\pi}\int_0^{2}\left(2r^3+\frac{8}{3}r-\frac{4}{3}r^4\right)dr\, d\theta = k\int_0^{2\pi}\left[\frac{2r^4}{4}+\frac{8}{3}\frac{r^2}{2}-\frac{4}{3}\cdot\frac{r^5}{5}\right]_0^2$$

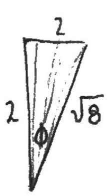
$$= k\int_0^{2\pi}\frac{24}{5}\, d\theta = \frac{24}{5}k\left[\theta\right]_0^{2\pi} = \boxed{\frac{48}{5}\pi k} \checkmark$$

B. T: $x = \rho\cos\theta\sin\phi \quad y = \rho\sin\theta\sin\phi \quad z = \rho\cos\phi \quad |J| = \rho^2\sin\phi$

$2 = \rho\cos\phi$

$\rho = \frac{2}{\cos\phi}$

$$m = k\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\frac{2}{\cos\phi}} (\rho^2)\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$



$\tan\phi = \frac{2}{2} = 1$

$\phi = \frac{\pi}{4}$

$$= k\int_0^{2\pi}\int_0^{\pi/4}\left[\frac{\rho^5}{5}\right]_0^{2/\cos\phi}\sin\phi\, d\phi\, d\theta = k\int_0^{2\pi}\int_0^{\pi/4}\frac{32}{5\cos^5\phi}\sin\phi\, d\phi\, d\theta$$

$u = \cos\phi \quad du = -\sin\phi\, d\phi$

$$= -\frac{32}{5}k\int_0^{2\pi}\left.\frac{u^{-4}}{-4}\right]_{u=1}^{u=\frac{\sqrt{2}}{2}} d\theta = \frac{8}{5}k\int_0^{2\pi}\left[\left(\frac{2}{\sqrt{2}}\right)^4-(1)^4\right] d\theta$$

$\frac{8}{4\sqrt{32}}$

$$= \frac{8}{5}k\int_0^{2\pi}\left(\frac{16}{4}-1\right)d\theta = \frac{24}{5}k\int_0^{2\pi}d\theta = \frac{24}{5}k(2\pi) = \boxed{\frac{48}{5}\pi k} \checkmark$$

DAY 3

② $\boxed{15.8 \# 24}$ ✓

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{3} e^{\rho}\rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$$

$$\int_0^{\pi/2} \sin\phi\, d\phi \int_0^{\pi/2} d\theta \int_0^{3} e^{\rho}\rho^2\, d\rho$$

$\rho^2 \quad + \quad e^{\rho}$
$2\rho \quad - \quad e^{\rho}$
$2 \quad + \quad e^{\rho}$
$0 \quad \quad e^{\rho}$

$$-\cos\phi\Big]_0^{\pi/2} \cdot \theta\Big]_0^{\pi/2} \cdot \Big[\rho^2 e^{\rho} - e^{\rho} 2\rho + 2e^{\rho}\Big]_0^{3}$$

$$= \frac{\pi}{2}\left[9e^3 - 6e^3 + 2e^3 - 2\right]$$

$$= \boxed{\frac{(5e^3 - 2)\pi}{2}} \quad ✓$$

⑥ $\boxed{15.8 \# 35}$ ✓

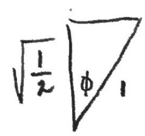
$x^2+y^2+z^2=1 \qquad z = \sqrt{x^2+y^2}$

$x^2+y^2+z^2=1 \qquad z^2 = x^2+y^2$

$2x^2+2y^2=1 \qquad z^2 = 1 - z^2$

$2z^2 = 1 \qquad z = \sqrt{\frac{1}{2}}$

$$\frac{x^2}{\left(\frac{1}{\sqrt{2}}\right)^2} + \frac{y^2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 1$$



$\cos\phi = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

$\phi = \pi/4$

$$\int_0^{\pi/4}\int_0^{2\pi}\int_0^{1} \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$

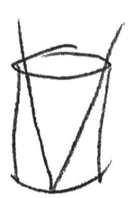
$$\int_0^{\pi/4} \sin\phi\, d\phi \cdot \int_0^{2\pi} d\theta \int_0^{1} \rho^2\, d\rho$$

$$= -\cos\phi\Big]_0^{\pi/4} \cdot 2\pi \; \frac{\rho^3}{3}\Big]_0^{1}$$

$$= \left[-\frac{\sqrt{2}}{2} + \frac{4}{4}\right] \cdot \frac{2\pi}{3} = \frac{2\pi}{3}\left(\frac{2-\sqrt{2}}{2}\right) = \frac{\pi}{3}(2-\sqrt{2}) \quad ✓$$

5 $\boxed{15.7 \ \#21}$

$$\int_0^{2\pi}\int_0^{1}\int_0^{2r} \cancel{z}\, r^3 \cos^2\theta \ dz\, dr\, d\theta$$

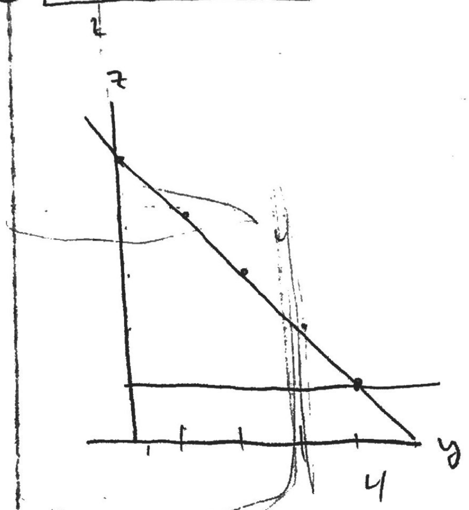


$$= \int_0^{2\pi}\int_0^{1} 2r^4\cos^2\theta \ dr\, d\theta$$

$$= 2\int_0^{2\pi} \cos^2\theta\, d\theta \cdot \int_0^{1} r^4\, dr$$

$$= 2\pi \cdot \frac{1}{5} = \frac{2\pi}{5}$$

3 $\boxed{15.6 \ \# 21}$



$$\int_0^{2\pi}\int_0^{3}\int_1^{5-r\sin\theta} r \ dz\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^{3} rz\Big]_1^{5-r\sin\theta} dr\, d\theta = \int_0^{2\pi}\int_0^{3}\left(4r - r^2\sin\theta\right) dr\, d\theta$$

$$= \int_0^{2\pi}\left[\frac{4r^2}{2} - \frac{r^3}{3}\sin\theta\right]_0^3 d\theta$$

$$= \int_0^{2\pi}\left[\frac{36}{2} - 9\sin\theta\right] d\theta$$

$$= \frac{36}{2}\theta + 9\cos\theta\Big]_0^{2\pi}$$

$$= 36\pi + 9 - 9$$

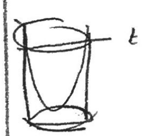
$$= 36\pi$$

4 $\boxed{15.7 \# 19}$

$$\int_0^{2\pi}\int_0^{\sqrt5}\int_0^{1+r^2} e^z\, r\, dz\, dr\, d\theta = \int_0^{2\pi} d\theta \int_0^{\sqrt5} r\, dr \int_0^{1+r^2} e^z\, dz$$

$$\frac{r^2}{2}$$

$$2\pi\left(\frac{5}{2}\right) e^z\Big]_0^{1+r^2}$$



$$\int_0^{2\pi}\int_0^{\sqrt5} e^z r\Big]_0^{1+r^2} dr\, d\theta = \int_0^{2\pi}\int_0^{\sqrt5}\left[e^{1+r^2}-1\right] r\, dr\, d\theta$$

$u = 1+r^2$

$du = 2r\, dr$

$dr = \frac{du}{2r}$

$$= \int_0^{2\pi}\int_0^{\sqrt5}\left(re^{1+r^2} - r\right) dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt5} re^{1+r^2}\, dr\, d\theta - \int_0^{2\pi}\int_0^{\sqrt5} r\, dr\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} e^{1+r^2}\Big|_0^{\sqrt5} d\theta \; -$$

$$= \frac{1}{2}\int_0^{2\pi}\left(e^6 - e\right) d\theta - \int_0^{2\pi} \frac{r^2}{2}\Big]_0^{\sqrt5} d\theta$$

$$= \frac{e^6 - e}{2}(2\pi) - \int_0^{2\pi} \frac{5}{2}\, d\theta$$

$$= \pi\left(e^6 - e\right) - 5\pi$$

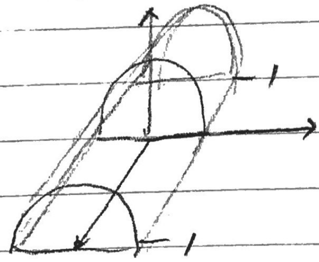
$$= \pi\left(e^6 - e - 5\right)$$

alternate on back

7 $\boxed{15.6 \ \#13.}$ $\iiint x^2 e^y \, dV$

$z = 1 - y^2$

$z = 0 \quad x = 1 \quad x = -1$



$$\int_{-1}^{1}\int_{-1}^{1}\int_{0}^{1-y^2} x^2 e^y \, dz \, dy \, dx$$

$$\int_{-1}^{1}\int_{-1}^{1} z x^2 e^y \Big]_0^{1-y^2} dy \, dx$$

$$\int_{-1}^{1}\int_{-1}^{1} (1-y^2) x^2 e^y \, dy \, dx$$

$$\int_{-1}^{1}\int_{-1}^{1} (x^2 e^y - y^2 e^y x^2) \, dy \, dx$$

$$\int_{-1}^{1} \left[x^2 e^y - x^2 [y^2 e^y - 2ye^y + 2e^y] \right]_{-1}^{1}$$

$$\int_{-1}^{1} \left(x^2 e - x^2 [e - 2e + 2e] \right) - \left(x^2 e^{-1} - x^2 [e^{-1} + 2e^{-1} + 2e^{-1}] \right)$$

$$\int_{-1}^{1} \cancel{x^2 e} - \cancel{x^2 e} + x^2(0) - x^2 e^{-1} + x^2 e^{-1} + x^2 2e^{-1} + x^2 2e^{-1}$$

$$\int_{-1}^{1} (3x^2 e^{-1}) \, dx = \frac{3}{e}\int_{-1}^{1} x^2 \, dx = \frac{1}{e} x^3 \Big]_{-1}^{1} = \frac{1}{e} + \frac{1}{e} = \frac{2}{e}$$

$(5e^{-1} \ldots e)$

$$\int_{-1}^{1} x^2 \, dx \cdot \int_{-1}^{1} (e^y - y^2 e^y) \, dy$$

$$\frac{x^3}{3}\Big]_{-1}^{1} \cdot \left[e^y - [y^2 e^y - 2ye^y + 2e^y] \right]_{-1}^{1}$$

$\frac{1}{3} + \frac{1}{3}$ $\qquad (\cancel{e} - [\cancel{e} - 2e + 2e]) - (e^{-1} - [e^{-1} + 2e^{-1} + 2e^{-1}])$

$\frac{2}{3}$ $\qquad \cancel{2e - 2e} \quad -e^{-1} + e^{-1} + 2e^{-1} + 2e^{-1}$

$$\frac{2}{3} \cdot \frac{4}{e} = \frac{+8}{3e}$$

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{-1}^{1} x^2 e^y \, dx\, dz\, dy \qquad \int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} \frac{x^3}{3} e^y \Big]_{-1}^{1} dz\, dy$$

$$= \int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} \frac{2}{3} e^y \, dz\, dy = \int_{-1}^{1} \frac{2}{3} e^y z \Big]_{0}^{\sqrt{1-y^2}} dy$$

$$\begin{array}{cc} y^2 & e^y \\ 2y & e^y \\ 2 & e^y \\ 0 & e^y \end{array}$$

$$= \frac{2}{3}\int_{-1}^{1} \left((1-y^2)e^y\right) dy = \frac{2}{3}\int_{-1}^{1} \left[e^y - y^2 e^y\right] dy$$

$$= \frac{2}{3}\left[e^y - y^2 e^y + 2ye^y - 2e^y\right]_{-1}^{1}$$

$$\int y^2 e^y \, dy = y^2 e^y - 2ye^y + 2e^y$$

$$= \frac{2}{3}\left[(e - e + 2e - 2e) - (e^{-1} - e^{-1} - 2e^{-1} - 2e^{-1})\right]$$

$$= \frac{2}{3}\left[4e^{-1}\right] = \frac{8e^{-1}}{3} \checkmark$$