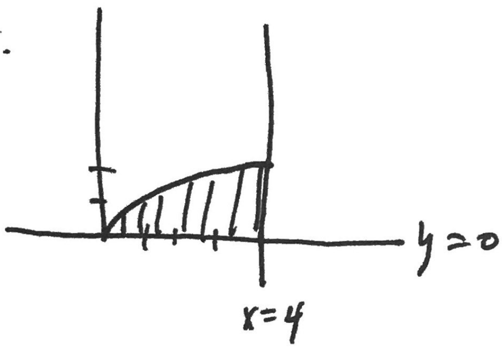
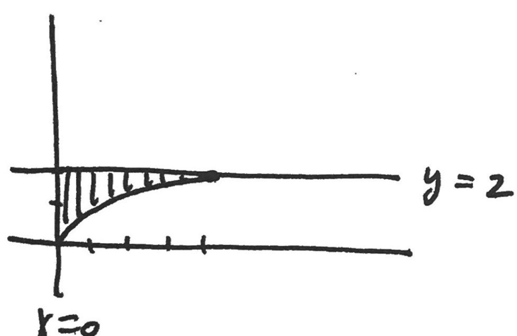
LESSON 1 PRACTICE SOLUTIONS

1.



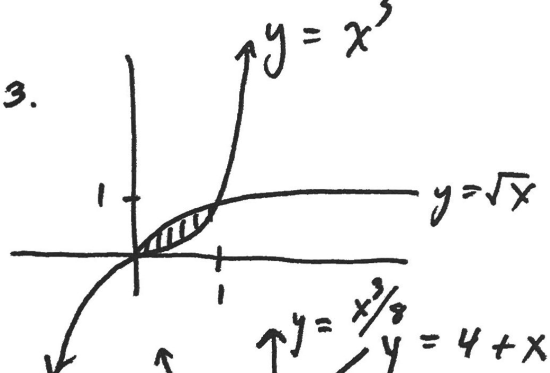
$$\int_0^4\int_0^{\sqrt{x}} f(x,y)\,dy\,dx = \int_0^2\int_{y^2}^4 f(x,y)\,dx\,dy$$

2.



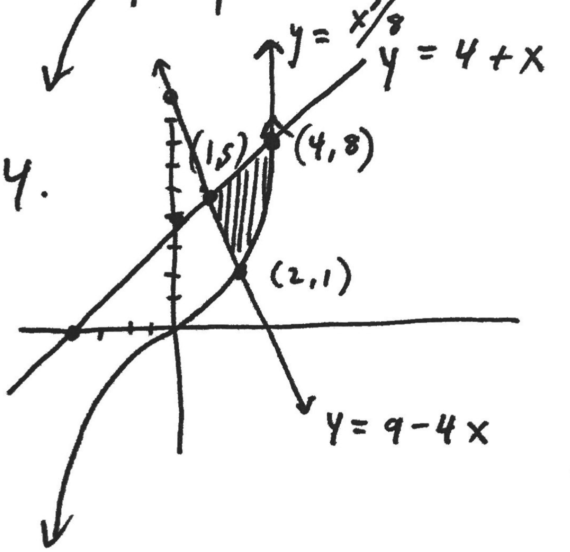
$$\int_0^4\int_{\sqrt{x}}^2 f(x,y)\,dy\,dx = \int_0^2\int_0^{y^2} f(x,y)\,dx\,dy$$

3.



$$\int_0^1\int_{x^3}^{\sqrt{x}} f(x,y)\,dy\,dx = \int_0^1\int_{y^2}^{\sqrt[3]{y}} f(x,y)\,dx\,dy$$

4.



$$\int_1^2\int_{9-4x}^{4+x} f(x,y)\,dy\,dx + \int_2^4\int_{x^3/8}^{4+x} f(x,y)\,dy\,dx$$

$$= \int_1^5\int_{\frac{y-9}{-4}}^{2\sqrt[3]{y}} f(x,y)\,dx\,dy + \int_5^8\int_{y-4}^{2\sqrt[3]{y}} f(x,y)\,dx\,dy$$

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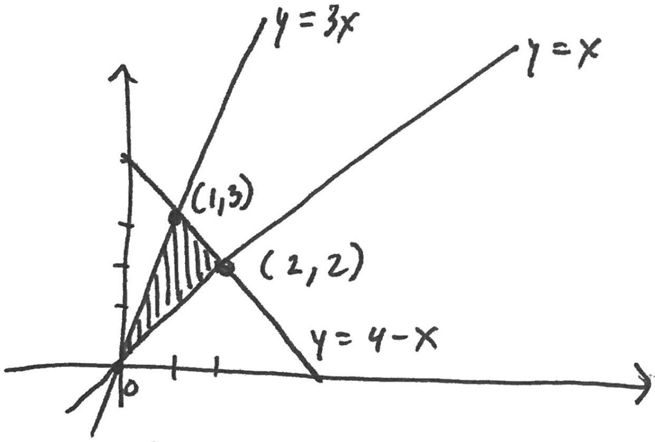

$$A = \int_0^1 \int_x^{3x} dy\, dx + \int_1^2 \int_x^{4-x} dy\, dx$$

$$= \int_0^1 y\Big]_x^{3x} dx + \int_1^2 y\Big]_x^{4-x} dx = \int_0^1 (3x - x)\, dx + \int_1^2 (4 - x - x)\, dx$$

$$\Rightarrow \cancel{} = \int_0^1 2x\, dx + \int_1^2 (4 - 2x)\, dx$$

$$= x^2\Big]_0^1 + 4x - x^2\Big]_1^2$$

$$= 1 + (8 - 4) - (4 - 1)$$

$$= 1 + 4 - 3 = \boxed{2}$$

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$$V = \int_0^1 \int_x^{3x} (x+y)\, dy\, dx + \int_1^2 \int_x^{4-x} (x+y)\, dy\, dx$$

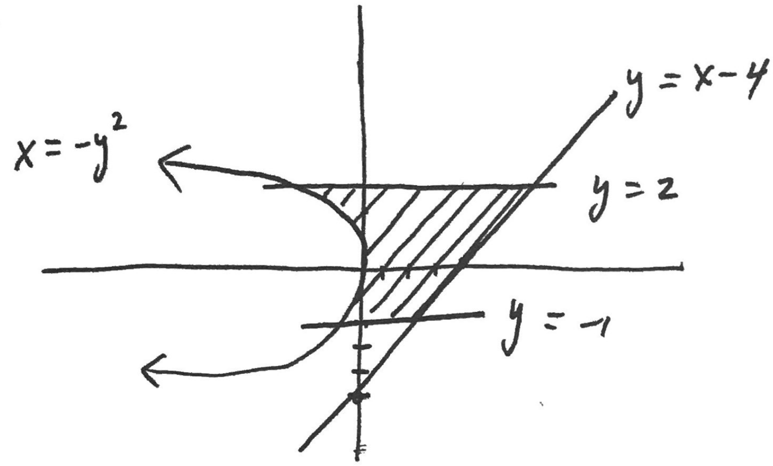
$$= \int_0^1 xy + \frac{y^2}{2}\Big]_x^{3x} dx + \int_1^2 xy + \frac{y^2}{2}\Big]_x^{4-x}$$

$$= \int_0^1 \left(3x^2 + \frac{9x^2}{2}\right) - \left(x^2 + \frac{x^2}{2}\right) dx + \int_1^2 4x - x^2 + \frac{(4-x)^2}{2} - \left(x^2 + \frac{x^2}{2}\right) dx$$

$$= \int_0^1 \frac{12}{2} x^2\, dx + \int_1^2 8 - 2x^2\, dx = \int_0^1 6x^2\, dx + \int_1^2 (8 - 2x^2)\, dx$$

$$= 2x^3\Big]_0^1 + \left[8x - \frac{2x^3}{3}\right]_1^2 = \underline{\underline{2}} + \left(16 - \frac{16}{3}\right) - \left(8 - \frac{2}{3}\right) = \boxed{\frac{16}{3}} \checkmark$$

7]



$$V = \int_{-1}^{2}\int_{-y^2}^{4+y} xy\,dx\,dy = \int_{-1}^{2} \frac{yx^2}{2}\Big]_{-y^2}^{4+y} dy$$

$$= \int_{-1}^{2}\left[\frac{y(4+y)^2}{2} - \frac{y(-y^2)^2}{2}\right] dy$$

$$= \int_{-1}^{2}\left\{\frac{y(16+8y+y^2)}{2} - \frac{y^5}{2}\right\} dy = \frac{1}{2}\int_{-1}^{2}(16y + 8y^2 + y^{3} - y^5)\,dy$$

$$= \frac{1}{2}\left[8y^2 + \frac{8y^3}{3} + \frac{y^4}{4} - \frac{y^6}{6}\right]_{-1}^{2}$$

$$= \frac{1}{2}\left[\left(32 + \frac{64}{3} + \overset{16}{\cancel{\frac{32}{4}}} - \frac{64}{6}\right) - \left(8 - \frac{8}{3} + \frac{1}{4} - \frac{1}{6}\right)\right]$$

$$= \cancel{\frac{1}{2}\left[\frac{68}{15} - \frac{65}{12}\right]} = \frac{1}{2}\left(\frac{140}{3} - \frac{65}{12}\right) = \frac{165}{8}$$

$$= \cancel{\frac{1}{2}\left[\frac{736}{15} - \frac{149}{30}\right] = \frac{1}{2}\left[\frac{441}{10}\right] = \frac{441}{20}}$$