49. (a) No. If $\mathbf{a}\cdot\mathbf{b}=\mathbf{a}\cdot\mathbf{c}$, then $\mathbf{a}\cdot(\mathbf{b}-\mathbf{c})=0$, so $\mathbf{a}$ is perpendicular to $\mathbf{b}-\mathbf{c}$, which can happen if $\mathbf{b}\neq\mathbf{c}$. For example, let $\mathbf{a}=\langle 1,1,1\rangle$, $\mathbf{b}=\langle 1,0,0\rangle$ and $\mathbf{c}=\langle 0,1,0\rangle$.

(b) No. If $\mathbf{a}\times\mathbf{b}=\mathbf{a}\times\mathbf{c}$ then $\mathbf{a}\times(\mathbf{b}-\mathbf{c})=\mathbf{0}$, which implies that $\mathbf{a}$ is parallel to $\mathbf{b}-\mathbf{c}$, which of course can happen if $\mathbf{b}\neq\mathbf{c}$.

(c) Yes. Since $\mathbf{a}\cdot\mathbf{c}=\mathbf{a}\cdot\mathbf{b}$, $\mathbf{a}$ is perpendicular to $\mathbf{b}-\mathbf{c}$, by part (a). From part (b), $\mathbf{a}$ is also parallel to $\mathbf{b}-\mathbf{c}$. Thus since $\mathbf{a}\neq\mathbf{0}$ but is both parallel and perpendicular to $\mathbf{b}-\mathbf{c}$, we have $\mathbf{b}-\mathbf{c}=\mathbf{0}$, so $\mathbf{b}=\mathbf{c}$.

13.5 Equations of Lines and Planes **ET 12.5**

1. (a) True; each of the first two lines has a direction vector parallel to the direction vector of the third line, so these vectors are each scalar multiples of the third direction vector. Then the first two direction vectors are also scalar multiples of each other, so these vectors, and hence the two lines, are parallel.

(b) False; for example, the x- and y-axes are both perpendicular to the z-axis, yet the x- and y-axes are not parallel.

(c) True; each of the first two planes has a normal vector parallel to the normal vector of the third plane, so these two normal vectors are parallel to each other and the planes are parallel.

(d) False; for example, the xy- and yz-planes are not parallel, yet they are both perpendicular to the xz-plane.

(e) False; the x- and y-axes are not parallel, yet they are both parallel to the plane $z=1$.

(f) True; if each line is perpendicular to a plane, then the lines' direction vectors are both parallel to a normal vector for the plane. Thus, the direction vectors are parallel to each other and the lines are parallel.

(g) False; the planes $y=1$ and $z=1$ are not parallel, yet they are both parallel to the x-axis.

(h) True; if each plane is perpendicular to a line, then any normal vector for each plane is parallel to a direction vector for the line. Thus, the normal vectors are parallel to each other and the planes are parallel.

(i) True; see Figure 9 and the accompanying discussion.

(j) False; they can be skew, as in Example 3.

(k) True. Consider any normal vector for the plane and any direction vector for the line. If the normal vector is perpendicular to the direction vector, the line and plane are parallel. Otherwise, the vectors meet at an angle θ, $0^\circ\leq\theta<90^\circ$, and the line will intersect the plane at an angle $90^\circ-\theta$.

3. For this line, we have $\mathbf{r}_0=2\,\mathbf{i}+2.4\,\mathbf{j}+3.5\,\mathbf{k}$ and $\mathbf{v}=3\,\mathbf{i}+2\,\mathbf{j}-\mathbf{k}$, so a vector equation is $\mathbf{r}=\mathbf{r}_0+t\,\mathbf{v}=(2\,\mathbf{i}+2.4\,\mathbf{j}+3.5\,\mathbf{k})+t(3\,\mathbf{i}+2\,\mathbf{j}-\mathbf{k})=(2+3t)\,\mathbf{i}+(2.4+2t)\,\mathbf{j}+(3.5-t)\,\mathbf{k}$ and parametric equations are $x=2+3t$, $y=2.4+2t$, $z=3.5-t$.

5. A line perpendicular to the given plane has the same direction as a normal vector to the plane, such as $\mathbf{n}=\langle 1,3,1\rangle$. So $\mathbf{r}_0=\mathbf{i}+6\,\mathbf{k}$, and we can take $\mathbf{v}=\mathbf{i}+3\,\mathbf{j}+\mathbf{k}$. Then a vector equation is $\mathbf{r}=(\mathbf{i}+6\,\mathbf{k})+t(\mathbf{i}+3\,\mathbf{j}+\mathbf{k})=(1+t)\,\mathbf{i}+3t\,\mathbf{j}+(6+t)\,\mathbf{k}$, and parametric equations are $x=1+t$, $y=3t$, $z=6+t$.

7. The vector $\mathbf{v}=\langle -4-1,3-3,0-2\rangle=\langle -5,0,-2\rangle$ is parallel to the line. Letting $P_0=(1,3,2)$, parametric equations are $x=1-5t$, $y=3+0t=3$, $z=2-2t$, while symmetric equations are $\dfrac{x-1}{-5}=\dfrac{z-2}{-2}$, $y=3$. Notice here that the direction number $b=0$, so rather than writing $\dfrac{y-3}{0}$ in the symmetric equation we must write the equation $y=3$ separately.

9. $\mathbf{v} = \left\langle 2-0, 1-\frac{1}{2}, -3-1 \right\rangle = \left\langle 2, \frac{1}{2}, -4 \right\rangle$, and letting $P_0 = (2, 1, -3)$, parametric equations are $x = 2+2t$, $y = 1+\frac{1}{2}t$, $z = -3-4t$, while symmetric equations are $\dfrac{x-2}{2} = \dfrac{y-1}{1/2} = \dfrac{z+3}{-4}$ or $\dfrac{x-2}{2} = 2y-2 = \dfrac{z+3}{-4}$.

11. The line has direction $\mathbf{v} = \langle 1, 2, 1 \rangle$. Letting $P_0 = (1, -1, 1)$, parametric equations are $x = 1+t$, $y = -1+2t$, $z = 1+t$ and symmetric equations are $x - 1 = \dfrac{y+1}{2} = z - 1$.

13. Direction vectors of the lines are $\mathbf{v}_1 = \langle -2-(-4), 0-(-6), -3-1 \rangle = \langle 2, 6, -4 \rangle$ and $\mathbf{v}_2 = \langle 5-10, 3-18, 14-4 \rangle = \langle -5, -15, 10 \rangle$, and since $\mathbf{v}_2 = -\frac{5}{2}\mathbf{v}_1$, the direction vectors and thus the lines are parallel.

15. (a) The line passes through the point $(1, -5, 6)$ and a direction vector for the line is $\langle -1, 2, -3 \rangle$, so symmetric equations for the line are $\dfrac{x-1}{-1} = \dfrac{y+5}{2} = \dfrac{z-6}{-3}$.

(b) The line intersects the xy-plane when $z = 0$, so we need $\dfrac{x-1}{-1} = \dfrac{y+5}{2} = \dfrac{0-6}{-3}$ or $\dfrac{x-1}{-1} = 2 \;\Rightarrow\; x = -1$, $\dfrac{y+5}{2} = 2 \;\Rightarrow\; y = -1$. Thus the point of intersection with the xy-plane is $(-1, -1, 0)$. Similarly for the yz-plane, we need $x = 0 \;\Rightarrow\; 1 = \dfrac{y+5}{2} = \dfrac{z-6}{-3} \;\Rightarrow\; y = -3$, $z = 3$. Thus the line intersects the yz-plane at $(0, -3, 3)$. For the xz-plane, we need $y = 0 \;\Rightarrow\; \dfrac{x-1}{-1} = \dfrac{5}{2} = \dfrac{z-6}{-3} \;\Rightarrow\; x = -\frac{3}{2}$, $z = -\frac{3}{2}$. So the line intersects the xz-plane at $\left(-\frac{3}{2}, 0, -\frac{3}{2}\right)$.

17. From Equation 4, the line segment from $\mathbf{r}_0 = 2\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}$ to $\mathbf{r}_1 = 4\,\mathbf{i} + 6\,\mathbf{j} + \mathbf{k}$ is $\mathbf{r}(t) = (1-t)\,\mathbf{r}_0 + t\,\mathbf{r}_1 = (1-t)(2\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}) + t(4\,\mathbf{i} + 6\,\mathbf{j} + \mathbf{k}) = (2\,\mathbf{i} - \mathbf{j} + 4\,\mathbf{k}) + t(2\,\mathbf{i} + 7\,\mathbf{j} - 3\,\mathbf{k})$, $0 \le t \le 1$.

19. Since the direction vectors are $\mathbf{v}_1 = \langle -6, 9, -3 \rangle$ and $\mathbf{v}_2 = \langle 2, -3, 1 \rangle$, we have $\mathbf{v}_1 = -3\mathbf{v}_2$ so the lines are parallel.

21. Since the direction vectors $\langle 1, 2, 3 \rangle$ and $\langle -4, -3, 2 \rangle$ are not scalar multiples of each other, the lines are not parallel, so we check to see if the lines intersect. The parametric equations of the lines are L_1: $x = t$, $y = 1+2t$, $z = 2+3t$ and L_2: $x = 3-4s$, $y = 2-3s$, $z = 1+2s$. For the lines to intersect, we must be able to find one value of t and one value of s that produce the same point from the respective parametric equations. Thus we need to satisfy the following three equations: $t = 3-4s$, $1+2t = 2-3s$, $2+3t = 1+2s$. Solving the first two equations we get $t = -1$, $s = 1$ and checking, we see that these values don't satisfy the third equation. Thus the lines aren't parallel and don't intersect, so they must be skew lines.

23. Since the plane is perpendicular to the vector $\langle -2, 1, 5 \rangle$, we can take $\langle -2, 1, 5 \rangle$ as a normal vector to the plane. $(6, 3, 2)$ is a point on the plane, so setting $a = -2$, $b = 1$, $c = 5$ and $x_0 = 6$, $y_0 = 3$, $z_0 = 2$ in Equation 7 gives $-2(x-6) + 1(y-3) + 5(z-2) = 0$ or $-2x + y + 5z = 1$ to be an equation of the plane.

25. $\mathbf{i} + \mathbf{j} - \mathbf{k} = \langle 1, 1, -1 \rangle$ is a normal vector to the plane and $(1, -1, 1)$ is a point on the plane, so setting $a = 1$, $b = 1$, $c = -1$, $x_0 = 1$, $y_0 = -1$, $z_0 = 1$ in Equation 7 gives $1\,(x-1) + 1[y-(-1)] - 1(z-1) = 0$ or $x + y - z = -1$ to be an equation of the plane.

27. Since the two planes are parallel, they will have the same normal vectors. So we can take $\mathbf{n} = \langle 2, -1, 3 \rangle$, and an equation of the plane is $2(x-0) - 1(y-0) + 3(z-0) = 0$ or $2x - y + 3z = 0$.

29. Since the two planes are parallel, they will have the same normal vectors. So we can take $\mathbf{n} = \langle 3, 0, -7 \rangle$, and an equation of the plane is $3(x-4) + 0[y-(-2)] - 7(z-3) = 0$ or $3x - 7z = -9$.

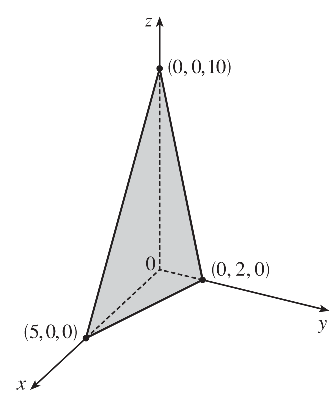
31. Here the vectors $\mathbf{a} = \langle 1-0, 0-1, 1-1 \rangle = \langle 1, -1, 0 \rangle$ and $\mathbf{b} = \langle 1-0, 1-1, 0-1 \rangle = \langle 1, 0, -1 \rangle$ lie in the plane, so $\mathbf{a} \times \mathbf{b}$ is a normal vector to the plane. Thus, we can take $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 1-0, 0+1, 0+1 \rangle = \langle 1, 1, 1 \rangle$. If P_0 is the point $(0, 1, 1)$, an equation of the plane is $1(x-0) + 1(y-1) + 1(z-1) = 0$ or $x + y + z = 2$.

33. Here the vectors $\mathbf{a} = \langle 8-3, 2-(-1), 4-2 \rangle = \langle 5, 3, 2 \rangle$ and $\mathbf{b} = \langle -1-3, -2-(-1), -3-2 \rangle = \langle -4, -1, -5 \rangle$ lie in the plane, so a normal vector to the plane is $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -15+2, -8+25, -5+12 \rangle = \langle -13, 17, 7 \rangle$ and an equation of the plane is $-13(x-3) + 17[y-(-1)] + 7(z-2) = 0$ or $-13x + 17y + 7z = -42$.

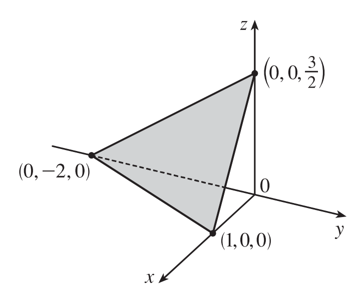
35. If we first find two nonparallel vectors in the plane, their cross product will be a normal vector to the plane. Since the given line lies in the plane, its direction vector $\mathbf{a} = \langle -2, 5, 4 \rangle$ is one vector in the plane. We can verify that the given point $(6, 0, -2)$ does not lie on this line, so to find another nonparallel vector $\mathbf{b}$ which lies in the plane, we can pick any point on the line and find a vector connecting the points. If we put $t = 0$, we see that $(4, 3, 7)$ is on the line, so $\mathbf{b} = \langle 6-4, 0-3, -2-7 \rangle = \langle 2, -3, -9 \rangle$ and $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -45+12, 8-18, 6-10 \rangle = \langle -33, -10, -4 \rangle$. Thus, an equation of the plane is $-33(x-6) - 10(y-0) - 4[z-(-2)] = 0$ or $33x + 10y + 4z = 190$.

37. A direction vector for the line of intersection is $\mathbf{a} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, -1 \rangle \times \langle 2, -1, 3 \rangle = \langle 2, -5, -3 \rangle$, and $\mathbf{a}$ is parallel to the desired plane. Another vector parallel to the plane is the vector connecting any point on the line of intersection to the given point $(-1, 2, 1)$ in the plane. Setting $x = 0$, the equations of the planes reduce to $y - z = 2$ and $-y + 3z = 1$ with simultaneous solution $y = \frac{7}{2}$ and $z = \frac{3}{2}$. So a point on the line is $\left(0, \frac{7}{2}, \frac{3}{2}\right)$ and another vector parallel to the plane is $\left\langle -1, -\frac{3}{2}, -\frac{1}{2} \right\rangle$. Then a normal vector to the plane is $\mathbf{n} = \langle 2, -5, -3 \rangle \times \left\langle -1, -\frac{3}{2}, -\frac{1}{2} \right\rangle = \langle -2, 4, -8 \rangle$ and an equation of the plane is $-2(x+1) + 4(y-2) - 8(z-1) = 0$ or $x - 2y + 4z = -1$.

39. To find the x-intercept we set $y = z = 0$ in the equation $2x + 5y + z = 10$ and obtain $2x = 10 \Rightarrow x = 5$ so the x-intercept is $(5, 0, 0)$. When $x = z = 0$ we get $5y = 10 \Rightarrow y = 2$, so the y-intercept is $(0, 2, 0)$. Setting $x = y = 0$ gives $z = 10$, so the z-intercept is $(0, 0, 10)$ and we graph the portion of the plane that lies in the first octant.



41. Setting $y = z = 0$ in the equation $6x - 3y + 4z = 6$ gives $6x = 6 \Rightarrow x = 1$, when $x = z = 0$ we have $-3y = 6 \Rightarrow y = -2$, and $x = y = 0$ implies $4z = 6 \Rightarrow z = \frac{3}{2}$, so the intercepts are $(1, 0, 0)$, $(0, -2, 0)$, and $\left(0, 0, \frac{3}{2}\right)$. The figure shows the portion of the plane cut off by the coordinate planes.



43. Substitute the parametric equations of the line into the equation of the plane: $(3-t)-(2+t)+2(5t)=9 \Rightarrow 8t=8 \Rightarrow t=1$. Therefore, the point of intersection of the line and the plane is given by $x=3-1=2$, $y=2+1=3$, and $z=5(1)=5$, that is, the point $(2,3,5)$.

45. Parametric equations for the line are $x=t$, $y=1+t$, $z=\frac{1}{2}t$ and substituting into the equation of the plane gives $4(t)-(1+t)+3\left(\frac{1}{2}t\right)=8 \Rightarrow \frac{9}{2}t=9 \Rightarrow t=2$. Thus $x=2$, $y=1+2=3$, $z=\frac{1}{2}(2)=1$ and the point of intersection is $(2,3,1)$.

47. Setting $x=0$, we see that $(0,1,0)$ satisfies the equations of both planes, so that they do in fact have a line of intersection. $\mathbf{v}=\mathbf{n}_1\times\mathbf{n}_2=\langle 1,1,1\rangle\times\langle 1,0,1\rangle=\langle 1,0,-1\rangle$ is the direction of this line. Therefore, direction numbers of the intersecting line are 1, 0, -1.

49. Normal vectors for the planes are $\mathbf{n}_1=\langle 1,4,-3\rangle$ and $\mathbf{n}_2=\langle -3,6,7\rangle$, so the normals (and thus the planes) aren't parallel. But $\mathbf{n}_1\cdot\mathbf{n}_2=-3+24-21=0$, so the normals (and thus the planes) are perpendicular.

51. Normal vectors for the planes are $\mathbf{n}_1=\langle 1,1,1\rangle$ and $\mathbf{n}_2=\langle 1,-1,1\rangle$. The normals are not parallel, so neither are the planes. Furthermore, $\mathbf{n}_1\cdot\mathbf{n}_2=1-1+1=1\neq 0$, so the planes aren't perpendicular. The angle between them is given by

$$\cos\theta=\frac{\mathbf{n}_1\cdot\mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|}=\frac{1}{\sqrt{3}\,\sqrt{3}}=\frac{1}{3} \quad\Rightarrow\quad \theta=\cos^{-1}\left(\tfrac{1}{3}\right)\approx 70.5^\circ.$$

53. The normals are $\mathbf{n}_1=\langle 1,-4,2\rangle$ and $\mathbf{n}_2=\langle 2,-8,4\rangle$. Since $\mathbf{n}_2=2\mathbf{n}_1$, the normals (and thus the planes) are parallel.

55. (a) To find a point on the line of intersection, set one of the variables equal to a constant, say $z=0$. (This will fail if the line of intersection does not cross the xy-plane; in that case, try setting x or y equal to 0.) The equations of the two planes reduce to $x+y=1$ and $x+2y=1$. Solving these two equations gives $x=1$, $y=0$. Thus a point on the line is $(1,0,0)$.

A vector $\mathbf{v}$ in the direction of this intersecting line is perpendicular to the normal vectors of both planes, so we can take $\mathbf{v}=\mathbf{n}_1\times\mathbf{n}_2=\langle 1,1,1\rangle\times\langle 1,2,2\rangle=\langle 2-2,1-2,2-1\rangle=\langle 0,-1,1\rangle$. By Equations 2, parametric equations for the line are $x=1$, $y=-t$, $z=t$.

(b) The angle between the planes satisfies $\cos\theta=\dfrac{\mathbf{n}_1\cdot\mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|}=\dfrac{1+2+2}{\sqrt{3}\,\sqrt{9}}=\dfrac{5}{3\sqrt{3}}$. Therefore $\theta=\cos^{-1}\left(\dfrac{5}{3\sqrt{3}}\right)\approx 15.8^\circ$.

57. Setting $z=0$, the equations of the two planes become $5x-2y=1$ and $4x+y=6$. Solving these two equations gives $x=1$, $y=2$ so a point on the line of intersection is $(1,2,0)$. A vector $\mathbf{v}$ in the direction of this intersecting line is perpendicular to the normal vectors of both planes. So we can use $\mathbf{v}=\mathbf{n}_1\times\mathbf{n}_2=\langle 5,-2,-2\rangle\times\langle 4,1,1\rangle=\langle 0,-13,13\rangle$ or equivalently we can take $\mathbf{v}=\langle 0,-1,1\rangle$, and symmetric equations for the line are $x=1$, $\dfrac{y-2}{-1}=\dfrac{z}{1}$ or $x=1$, $y-2=-z$.

59. The distance from a point (x,y,z) to $(1,0,-2)$ is $d_1=\sqrt{(x-1)^2+y^2+(z+2)^2}$ and the distance from (x,y,z) to $(3,4,0)$ is $\sqrt{(x-3)^2+(y-4)^2+z^2}$. The plane consists of all points (x,y,z) where $d_1=d_2 \Rightarrow d_1^2=d_2^2 \Leftrightarrow$

$(x-1)^2+y^2+(z+2)^2=(x-3)^2+(y-4)^2+z^2 \Leftrightarrow$

$x^2-2x+y^2+z^2+4z+5=x^2-6x+y^2-8y+z^2+25 \Leftrightarrow 4x+8y+4z=20$ so an equation for the plane is $4x+8y+4z=20$ or equivalently $x+2y+z=5$.

Alternatively, you can argue that the segment joining points $(1,0,-2)$ and $(3,4,0)$ is perpendicular to the plane and the plane includes the midpoint of the segment.

61. The plane contains the points $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$. Thus the vectors $\mathbf{a} = \langle -a, b, 0 \rangle$ and $\mathbf{b} = \langle -a, 0, c \rangle$ lie in the plane, and $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle bc - 0, 0 + ac, 0 + ab \rangle = \langle bc, ac, ab \rangle$ is a normal vector to the plane. The equation of the plane is therefore $bcx + acy + abz = abc + 0 + 0$ or $bcx + acy + abz = abc$. Notice that if $a \neq 0$, $b \neq 0$ and $c \neq 0$ then we can rewrite the equation as $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$. This is a good equation to remember!

63. Two vectors which are perpendicular to the required line are the normal of the given plane, $\langle 1, 1, 1 \rangle$, and a direction vector for the given line, $\langle 1, -1, 2 \rangle$. So a direction vector for the required line is $\langle 1, 1, 1 \rangle \times \langle 1, -1, 2 \rangle = \langle 3, -1, -2 \rangle$. Thus L is given by $\langle x, y, z \rangle = \langle 0, 1, 2 \rangle + t\langle 3, -1, -2 \rangle$, or in parametric form, $x = 3t$, $y = 1 - t$, $z = 2 - 2t$.

65. Let P_i have normal vector $\mathbf{n}_i$. Then $\mathbf{n}_1 = \langle 4, -2, 6 \rangle$, $\mathbf{n}_2 = \langle 4, -2, -2 \rangle$, $\mathbf{n}_3 = \langle -6, 3, -9 \rangle$, $\mathbf{n}_4 = \langle 2, -1, -1 \rangle$. Now $\mathbf{n}_1 = -\frac{2}{3}\mathbf{n}_3$, so $\mathbf{n}_1$ and $\mathbf{n}_3$ are parallel, and hence P_1 and P_3 are parallel; similarly P_2 and P_4 are parallel because $\mathbf{n}_2 = 2\mathbf{n}_4$. However, $\mathbf{n}_1$ and $\mathbf{n}_2$ are not parallel. $\left(0, 0, \frac{1}{2}\right)$ lies on P_1, but not on P_3, so they are not the same plane, but both P_2 and P_4 contain the point $(0, 0, -3)$, so these two planes are identical.

67. Let $Q = (1, 3, 4)$ and $R = (2, 1, 1)$, points on the line corresponding to $t = 0$ and $t = 1$. Let $P = (4, 1, -2)$. Then $\mathbf{a} = \overrightarrow{QR} = \langle 1, -2, -3 \rangle$, $\mathbf{b} = \overrightarrow{QP} = \langle 3, -2, -6 \rangle$. The distance is

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|} = \frac{|\langle 1, -2, -3 \rangle \times \langle 3, -2, -6 \rangle|}{|\langle 1, -2, -3 \rangle|} = \frac{|\langle 6, -3, 4 \rangle|}{|\langle 1, -2, -3 \rangle|} = \frac{\sqrt{6^2 + (-3)^2 + 4^2}}{\sqrt{1^2 + (-2)^2 + (-3)^2}} = \frac{\sqrt{61}}{\sqrt{14}} = \sqrt{\frac{64}{14}}.$$

69. By Equation 9, the distance is $D = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} = \dfrac{|3(1) + 2(-2) + 6(4) - 5|}{\sqrt{3^2 + 2^2 + 6^2}} = \dfrac{|18|}{\sqrt{49}} = \dfrac{18}{7}$.

71. Put $y = z = 0$ in the equation of the first plane to get the point $(2, 0, 0)$ on the plane. Because the planes are parallel, the distance D between them is the distance from $(2, 0, 0)$ to the second plane. By Equation 9,

$$D = \frac{|4(2) - 6(0) + 2(0) - 3|}{\sqrt{4^2 + (-6)^2 + (2)^2}} = \frac{5}{\sqrt{56}} = \frac{5}{2\sqrt{14}} \text{ or } \frac{5\sqrt{14}}{28}.$$

73. The distance between two parallel planes is the same as the distance between a point on one of the planes and the other plane. Let $P_0 = (x_0, y_0, z_0)$ be a point on the plane given by $ax + by + cz + d_1 = 0$. Then $ax_0 + by_0 + cz_0 + d_1 = 0$ and the distance between P_0 and the plane given by $ax + by + cz + d_2 = 0$ is, from Equation 9,

$$D = \frac{|ax_0 + by_0 + cz_0 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|-d_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}.$$

75. L_1: $x = y = z \Rightarrow x = y$ (**1**). L_2: $x + 1 = y/2 = z/3 \Rightarrow x + 1 = y/2$ (**2**). The solution of (**1**) and (**2**) is $x = y = -2$. However, when $x = -2$, $x = z \Rightarrow z = -2$, but $x + 1 = z/3 \Rightarrow z = -3$, a contradiction. Hence the lines do not intersect. For L_1, $\mathbf{v}_1 = \langle 1, 1, 1 \rangle$, and for L_2, $\mathbf{v}_2 = \langle 1, 2, 3 \rangle$, so the lines are not parallel. Thus the lines are skew lines. If two lines are skew, they can be viewed as lying in two parallel planes and so the distance between the skew lines would be the same as the distance between these parallel planes. The common normal vector to the planes must be perpendicular to both $\langle 1, 1, 1 \rangle$ and $\langle 1, 2, 3 \rangle$, the direction vectors of the two lines. So set

$\mathbf{n} = \langle 1, 1, 1 \rangle \times \langle 1, 2, 3 \rangle = \langle 3 - 2, -3 + 1, 2 - 1 \rangle = \langle 1, -2, 1 \rangle$. From above, we know that $(-2, -2, -2)$ and $(-2, -2, -3)$ are points of L_1 and L_2 respectively. So in the notation of Equation 8, $1(-2) - 2(-2) + 1(-2) + d_1 = 0 \quad \Rightarrow \quad d_1 = 0$ and $1(-2) - 2(-2) + 1(-3) + d_2 = 0 \quad \Rightarrow \quad d_2 = 1$.

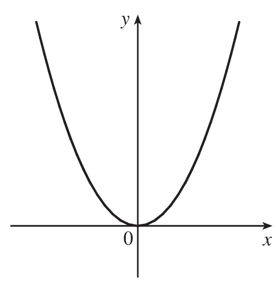
By Exercise 73, the distance between these two skew lines is $D = \dfrac{|0 - 1|}{\sqrt{1 + 4 + 1}} = \dfrac{1}{\sqrt{6}}$.

Alternate solution (without reference to planes): A vector which is perpendicular to both of the lines is $\mathbf{n} = \langle 1, 1, 1 \rangle \times \langle 1, 2, 3 \rangle = \langle 1, -2, 1 \rangle$. Pick any point on each of the lines, say $(-2, -2, -2)$ and $(-2, -2, -3)$, and form the vector $\mathbf{b} = \langle 0, 0, 1 \rangle$ connecting the two points. The distance between the two skew lines is the absolute value of the scalar projection of $\mathbf{b}$ along $\mathbf{n}$, that is, $D = \dfrac{|\mathbf{n} \cdot \mathbf{b}|}{|\mathbf{n}|} = \dfrac{|1 \cdot 0 - 2 \cdot 0 + 1 \cdot 1|}{\sqrt{1 + 4 + 1}} = \dfrac{1}{\sqrt{6}}$.

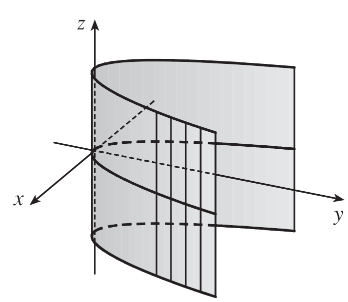
77. If $a \neq 0$, then $ax + by + cz + d = 0 \quad \Rightarrow \quad a(x + d/a) + b(y - 0) + c(z - 0) = 0$ which by (7) is the scalar equation of the plane through the point $(-d/a, 0, 0)$ with normal vector $\langle a, b, c \rangle$. Similarly, if $b \neq 0$ (or if $c \neq 0$) the equation of the plane can be rewritten as $a(x - 0) + b(y + d/b) + c(z - 0) = 0$ [or as $a(x - 0) + b(y - 0) + c(z + d/c) = 0$] which by (7) is the scalar equation of a plane through the point $(0, -d/b, 0)$ [or the point $(0, 0, -d/c)$] with normal vector $\langle a, b, c \rangle$.

13.6 Cylinders and Quadric Surfaces — ET 12.6

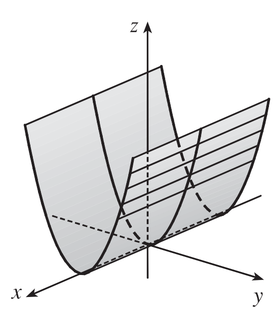
1. (a) In $\mathbb{R}^2$, the equation $y = x^2$ represents a parabola.



(b) In $\mathbb{R}^3$, the equation $y = x^2$ doesn't involve z, so any horizontal plane with equation $z = k$ intersects the graph in a curve with equation $y = x^2$. Thus, the surface is a parabolic cylinder, made up of infinitely many shifted copies of the same parabola. The rulings are parallel to the z-axis.



(c) In $\mathbb{R}^3$, the equation $z = y^2$ also represents a parabolic cylinder. Since x doesn't appear, the graph is formed by moving the parabola $z = y^2$ in the direction of the x-axis. Thus, the rulings of the cylinder are parallel to the x-axis.



LESSON 2 EVEN HW ANSWERS + some additional odd solutions

2. $\vec{r}(t) = \langle 6+t, -5+3t, 2-\frac{2}{3}t \rangle \quad t \in \mathbb{R}$

26. $x + 2y - 3z = -16$

46. $P(1,0,1)\ Q(4,-2,2)$

$\vec{v} = \langle 3,-2,1 \rangle$

$\vec{r}(t) = \langle 1+3t, -2t, 1+t \rangle$

$x + y + z = 6$

$(1+3t) - 2t + (1+t) = 6$

$t = 2$

Answer: $(7,-4,3)$

48. $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{1+2+3}{\sqrt{3}\sqrt{14}} = \dfrac{6}{\sqrt{42}}$

54. $\langle 1,2,2 \rangle$ vs $\langle 2,-1,2 \rangle$

$\vec{a}\cdot\vec{b} = 2 - 2 + 4 = 4$ (NOT ORTHOGONAL)

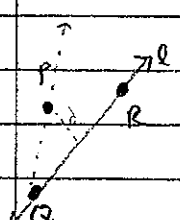
$\vec{a} \neq k\vec{b}$ (NOT PARALLEL)

$\theta = \cos^{-1}\dfrac{4}{\sqrt{9}\sqrt{9}} \qquad \theta = \cos^{-1}\dfrac{4}{9}$

67. $d = \dfrac{|\vec{QR} \times \vec{QP}|}{|\vec{QR}|}$ b/c $|\vec{QR} \times \vec{QP}| = |QR|\cdot d$

$t=0 \rightarrow$ pt $(1,3,4) \leftarrow Q \qquad P \rightarrow (4,1,-2)$

$t=1 \rightarrow$ pt $(2,1,1) \leftarrow R$



$\vec{QR} = \langle 1,-2,-3 \rangle$

$\vec{QP} = \langle 3,-2,-6 \rangle$

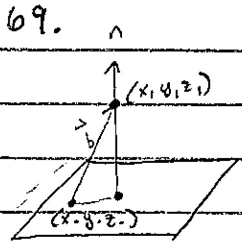
$\vec{QR} \times \vec{QP} = \langle 6,-3,4 \rangle$

$|\vec{QR} \times \vec{QP}| = \sqrt{36+9+16} = \sqrt{61}$

$|\vec{QR}| = \sqrt{1+4+9} = \sqrt{14}$

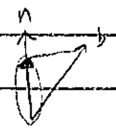
$d = \dfrac{|\vec{QR} \times \vec{QP}|}{|\vec{QR}|} = \dfrac{\sqrt{61}}{\sqrt{14}}$

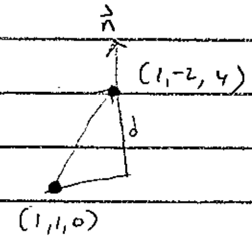
69.



$$d = \left|\text{Comp}_n \vec{b}\right| = \left|\frac{\vec{n}\cdot\vec{b}}{|\vec{n}|}\right|$$

Pt $(1,-2,4)$ $\quad \vec{n} = \langle 3, 2, 6\rangle$ $\quad$ Point on plane: $(1,1,0)$ (I PICK)





$$\vec{b} = \langle 0, -3, 4\rangle$$

$$d = \left|\frac{-6+24}{\sqrt{9+4+36}}\right| = \boxed{\frac{18}{7}}$$

— — — — [OR] — — — —

Pt $(1,-2,4)$ $\quad \vec{n} = \langle 3,2,6\rangle$ $\quad \vec{b} = \langle 1-x_0, -2-y_0, 4-z_0\rangle$

$$d = \left|\frac{3(1-x_0)+2(-2-y_0)+6(4-z_0)}{\sqrt{9+4+36}}\right|$$

$$= \frac{3-4+24-\left[3(x_0)+2(y_0)+6(z_0)\right]}{7}$$

$$= \frac{23-(5)}{7} = \boxed{\frac{18}{7}}$$

— — — — — — — — — — — — — — — —

EQUATION 9

$$d = \left|\frac{ax_1+by_1+cz_1-\overbrace{(ax_0+by_0+cz_0)}^{d}}{\sqrt{a^2+b^2+c^2}}\right|$$

where plane is $ax+by+cz=d$

point is (x_1,y_1,z_1)

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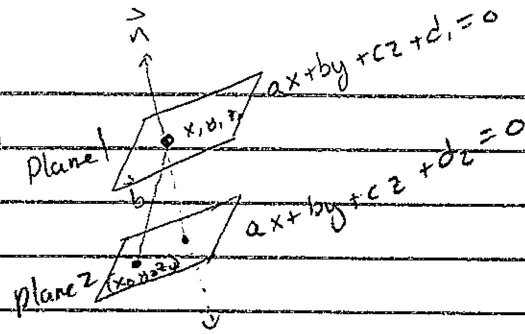

$\vec{n} = \langle a, b, c \rangle$

pt on plane 1 (x_1, y_1, z_1)

pt on plane 2 : $(x_0\ y_0\ z_0)$

$$\vec{b} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

$$\vec{n} = \langle a, b, c \rangle$$

$$d = \left| \frac{\vec{n} \cdot \vec{b}}{|\vec{n}|} \right| = \left| \frac{a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)}{\sqrt{a^2+b^2+c^2}} \right|$$

$$= \left| \frac{ax_1 + by_1 + cz_1 - [a(x_0) + b(y_0) + c(z_0)]}{\sqrt{a^2+b^2+c^2}} \right|$$

$$= \frac{|-d_1 - (-d_2)|}{\sqrt{a^2+b^2+c^2}} = \frac{|-d_1 + d_2|}{\sqrt{a^2+b^2+c^2}} = \frac{|d_1 - d_2|}{\sqrt{a^2+b^2+c^2}}$$

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Plane 1: $2x - 3y + z - 4 = 0 \rightarrow 2x - 3y + z - 4 = 0$

plane 2: $4x - 6y + 2z - 3 = 0 \rightarrow 2x - 3y + z - 3/2 = 0$

$$d = \frac{|-4 - (-3/2)|}{\sqrt{2^2 + (-3)^2 + (1)^2}} = \frac{|-8/2 + 3/2|}{\sqrt{4+9+1}} = \frac{5/2}{\sqrt{14}} = \boxed{\frac{5}{2\sqrt{14}}}$$