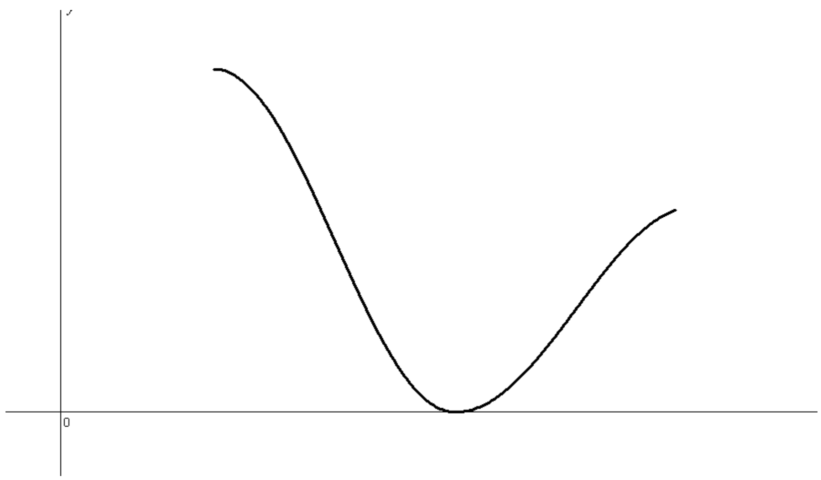
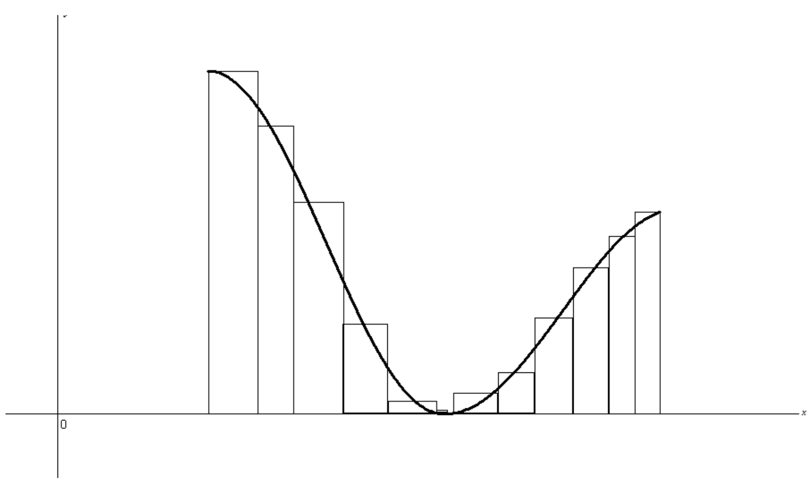
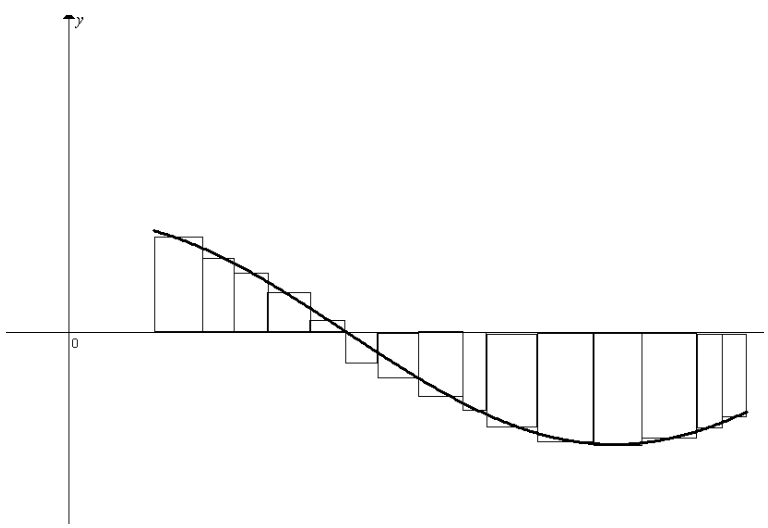
BC Q301 CH5: LESSON 1A AREA and INTEGRAL CONNECTION
Area – Integral Connection and Riemann Sums







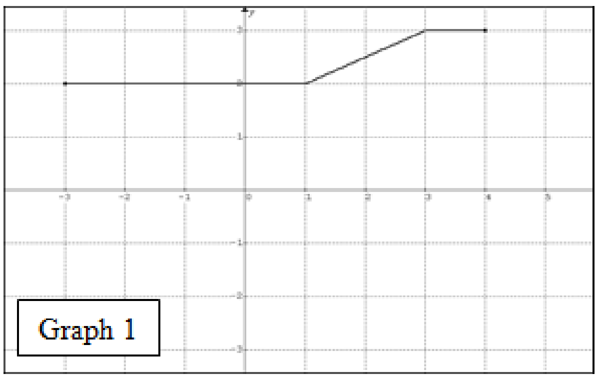
I. INTEGRAL AND AREA – BY HAND (APPEAL TO GEOMETRY)

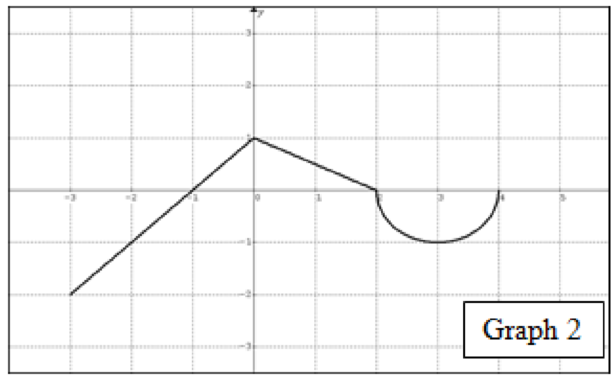
NOTES: Below are graphs that each represent a different $f(x)$ from $x = -3$ to $x = 4$.

A] Find the Area bounded by the graph of f and the x-axis.

B] Evaluate the integral $\int_{-3}^{4} f(x)dx$. Express the integral as it relates to a collection of areas.

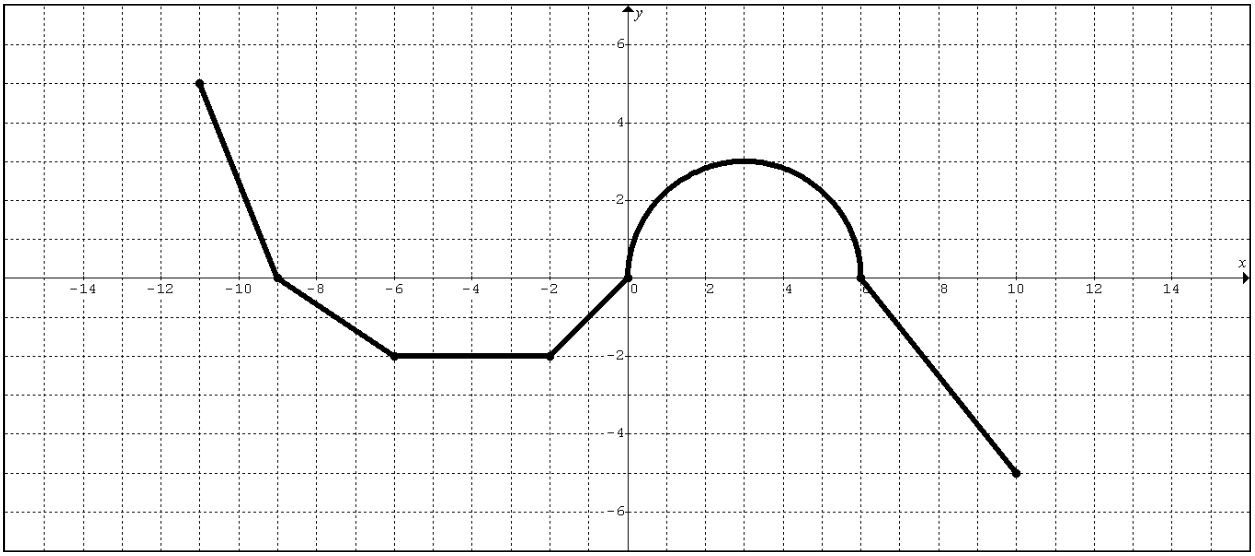
C] Express the Area as it relates to an (or set of) integral(s).





PRACTICE

1. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

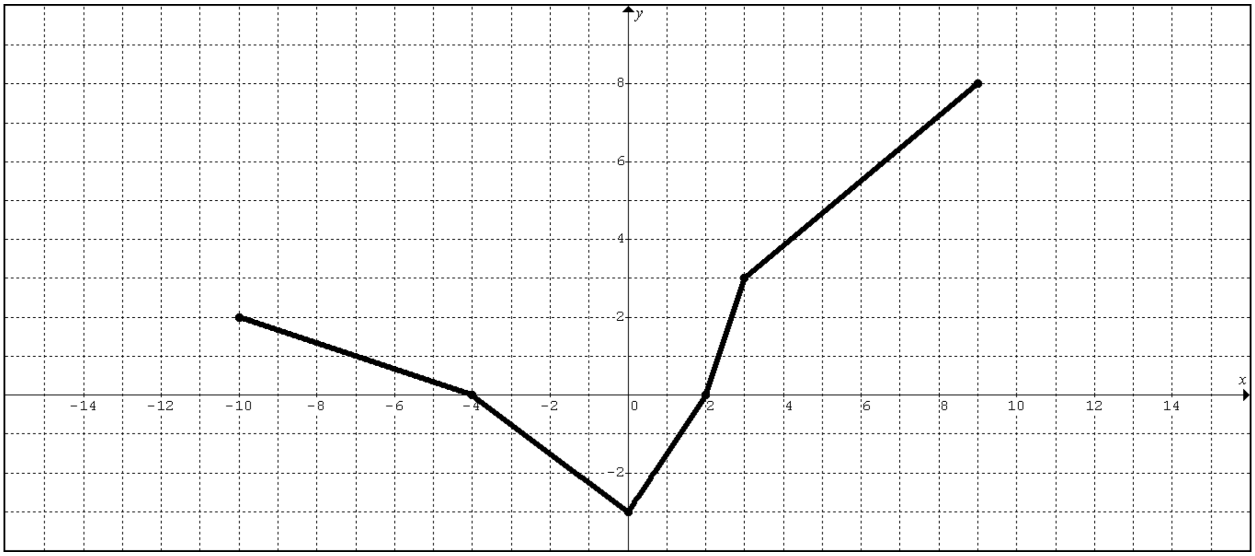
B. $\int_{-11}^{10} f(x)dx$

C. $\int_{0}^{10} f(x)dx$

D. $\int_{-9}^{-2} f(x)dx$

E. $\int_{0}^{-9} f(x)dx$

2. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

B. $\int_{-10}^{9} f(x)dx$

C. $\int_{-4}^{2} f(x)dx$

D. $\int_{-4}^{-10} f(x)dx$

E. $\int_{2}^{2} f(x)dx$

3. Evaluate each integral by appealing to geometry.

A. $\int_{-2}^{4}(1-x)dx$

B. $\int_{-2}^{4}2dx$

II. CH5 – (INTEGRAL PROPERTIES):

Suppose that *f* and *h* are continuous functions and that $\int_1^9 f(x)dx = -1$, $\int_7^9 f(x)dx = 5$, and $\int_7^9 h(x)dx = 4$.

Find each integral below:

$$\int_1^9 -2f(x)dx$$

$$\int_7^9 [f(x) + h(x)]dx$$

$$\int_7^9 [2f(x) - 3h(x)]dx$$

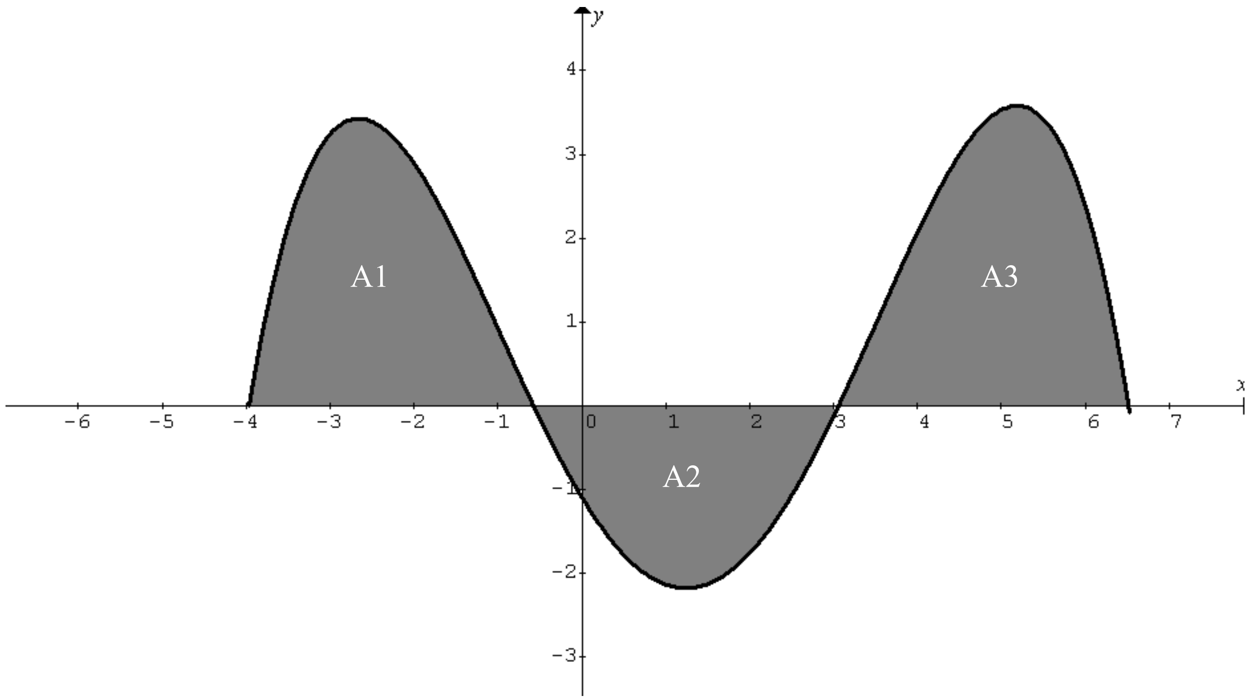
$$\int_9^1 f(x)dx$$

$$\int_1^7 f(x)dx$$

$$\int_9^7 [h(x) - f(x)]dx$$

BC: Q301 CH5A – LESSON 1A HOMEWORK

1.



In the diagram above, the values of the areas A1, A2, and A3 bounded by the graph of $f(x)$ and the *x*-axis, are 7, 5, and 8 square units respectively. $f(x)$ has zeros at -4, -0.6, 3, and 6.5.

Calculate the following definite integrals:

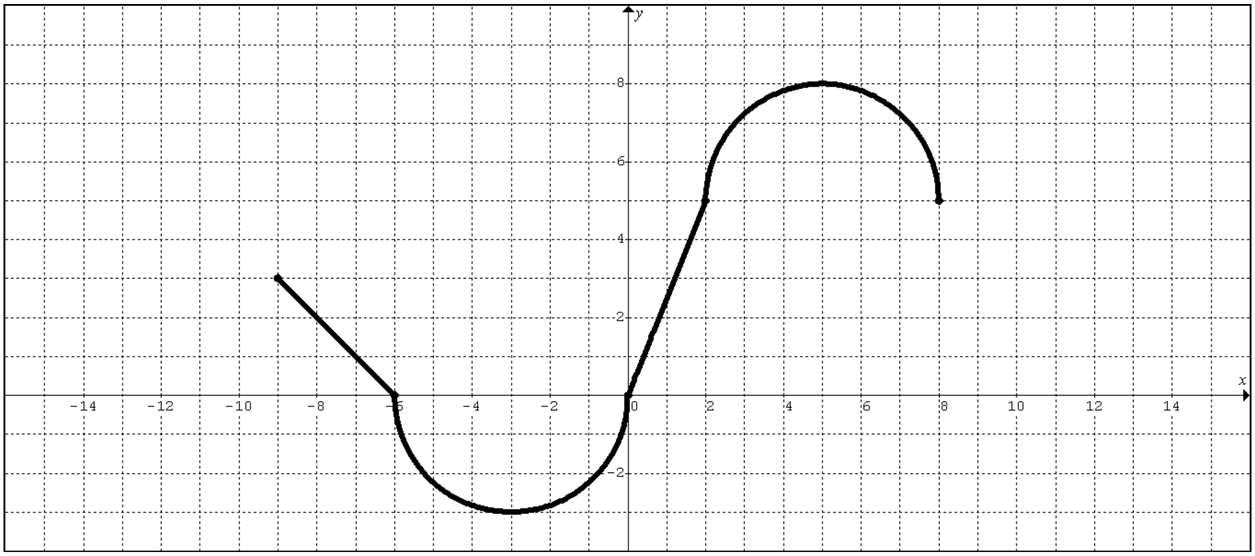
A. $\int_{-0.6}^{3} f(x)dx$

B. $\int_{-4}^{6.5} f(x)dx$

C. $\int_{3}^{-4} f(x)dx$

D. $\int_{-4}^{-0.6} f(x)dx - \int_{-0.6}^{3} f(x)dx + \int_{3}^{6.5} f(x)dx$

2. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

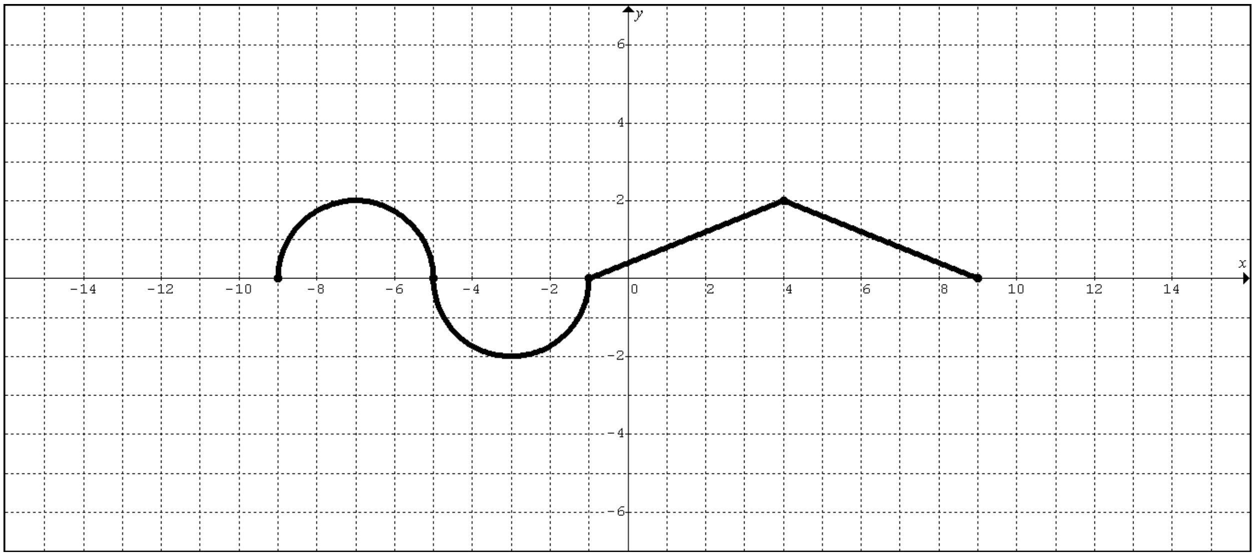
B. $\int_{-9}^{8} f(x)dx$

C. $\int_{-3}^{2} f(x)dx$

D. $\int_{0}^{-9} f(x)dx$

E. $7+\int_{-8}^{-6} f(x)dx$

3. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

B. $\int_{-9}^{9} f(x)dx$

C. $\int_{-9}^{-1} f(x)dx$

D. $\int_{-3}^{4} f(x)dx$

E. $10 - \int_{2}^{2} f(x)dx$

4. Evaluate the following by appealing to geometry.

A. $\int_{-2}^{1} (x-2)dx$

B. $\int_{-2}^{6} -4dx$

5. Suppose that *f* and *g* are continuous functions and that …

$\int_{1}^{2} f(x)dx = -4$, $\int_{1}^{5} f(x)dx = 6$, and $\int_{1}^{5} g(x)dx = 8$

Evaluate each of the following integrals:

A. $\int_{2}^{2} g(x)dx$

B. $\int_{5}^{1} g(x)dx$

C. $\int_{1}^{2} 3f(x)dx$

D. $\int_{2}^{5} f(x)dx$

E. $\int_{1}^{5} [f(x)-g(x)]dx$

F. $\int_{1}^{5} [4f(x)-g(x)]dx$

BC Q301 CH5: LESSON 1B AREA/INTEGRAL/FTC2

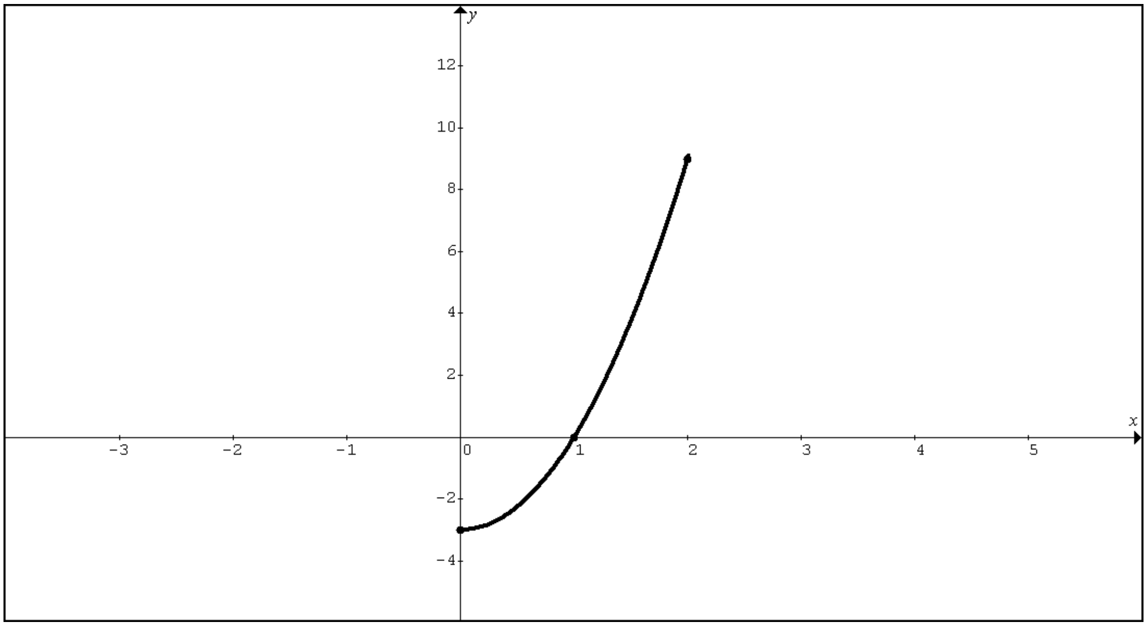
FTC2: Fundamental Theorem of Calculus (Part II) – Evaluation Method

I. [**FTC2-INTEGRAL**] Evaluate the following integrals (a) BY HAND and (b) BY TI89

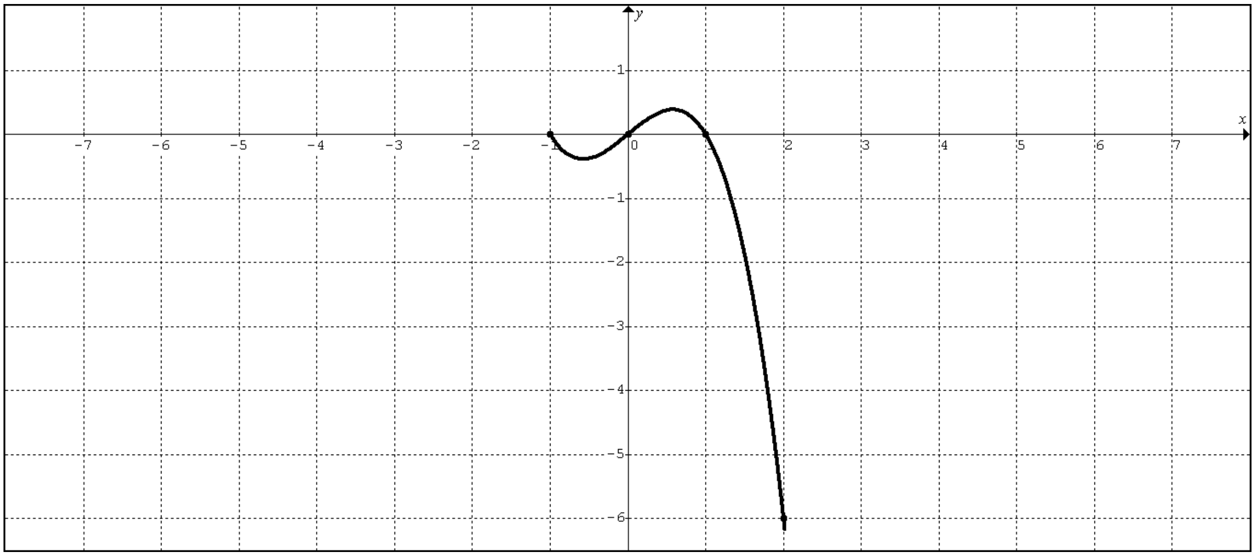
II. [**FTC2- AREA**]

1. Find the area bounded by $y = 3x^2 - 3$ and the x-axis on the interval [0, 2]

(a) BY HAND (b) BY TI89



2. Find the area bounded by $y = x - x^3$ and the x-axis on the interval [-1, 2].
(a) BY HAND (b) BY TI89



III. [**FTC2-AVERAGE VALUE**]:

1. (No Calculator) Find the average value of $f(x) = 3x^2 - 3$ on [0, 2]

2. (No Calculator) Find the average value of $y = x - x^3$ on [-1, 2]

IV. [**FTC2-UB**]: (No Calculator)
Rewrite and evaluate each integral using the appropriate u-substitution and u-bounds.

1. $\int_{-1}^{2} 45x^2\left(5x^3+1\right)^2 dx$

2. $\int_{0}^{\pi} \sin(5x)dx$

V. [**FTC2–ID**]: (Technology Required) Fundamental Theorem of Calculus "in disguise"

1. Suppose $f'(x) = e^{-x^2}$ and $f(1) = 3$. Find $f(2)$.

2. Suppose $f'(x) = (e^x)^{\cos(x)}$ and $f(2) = 1$. Find $f(0)$.

BC Q301 CH5A LESSON 1B HW:

FTC2 INTEGRAL– BY HAND:
Section 5.3: 21, 23, 25, 27
Section 5.4: 27, 29, 31, 33, 35, 37

FTC2 INTEGRAL– BY TI89:
Section 5.4: 49, 50

FTC2 AREA– BY TI89:
Section 5.4: 41, 43, 45, 47

(APPEAL TO GEOMETRY) AVERAGE VALUE – BY HAND:
Section 5.3: 15, 16

FTC2 AVERAGE VALUE – BY HAND:
Section 5.3: 32, 34, 35

FTC2 AVERAGE VALUE – BY TI89:
Section 5.3: 11, 13

FTC2-UB

1. Rewrite and evaluate $\int_{\frac{3}{4}\pi}^{\frac{\pi}{2}} \cos(2x)\, dx$ using an appropriate u-substitution and u-bounds.
2. Rewrite and evaluate $\int_{1}^{2} (2x-1)^7\, dx$ using an appropriate u-substitution and u-bounds.

FTC2-ID

1. Suppose $f'(x) = (2+\sin x)^x$ and $f(-1) = 5$. Find $f(4)$
2. Suppose $f'(x) = (2+\ln x)^x$ and $f(2) = 5$. Find $f(0.5)$

BC.Q301: LESSON 2 – FTC2: Fundamental Theorem of Calculus (Part II)
Lesson 2A: APPLICATIONS

A. Area Connection (Lesson 1-A)

B. Evaluation Method [FTC2] (Lesson 1-B)

C. FTC2 APPLICATIONS

C1: **Displacement/Position/Total Distance (Lesson 2A)**

C1: Displacement/Position/Total Distance (Lesson 2A)

A particle moves along the x-axis such that its position at time t is given as $x(t)$.
The velocity of the particle at time t is given as $x'(t) = v(t)$.

Review:

- The particle is moving to the right when $v(t) > 0$
- Acceleration is positive when $v'(t) > 0$
- The particle is getting faster (speed increasing) when $v(t)$ and $a(t)$ share the same sign.

New:

LESSON 2A NOTES

1. An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

A. What is the total distance traveled by the object over the time period $0 \leq t \leq 4$?
B. What is the position of the object at time $t = 4$?
C. What is the average velocity over the time period $0 \leq t \leq 4$?
D. What is the average acceleration over the time period $0 \leq t \leq 4$?

LESSON 2A NOTES

2. A particle moves along the *x*-axis so that its velocity at time *t* is given as

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

A. What is the total distance traveled by the particle from time t = 0 until time t = 3?

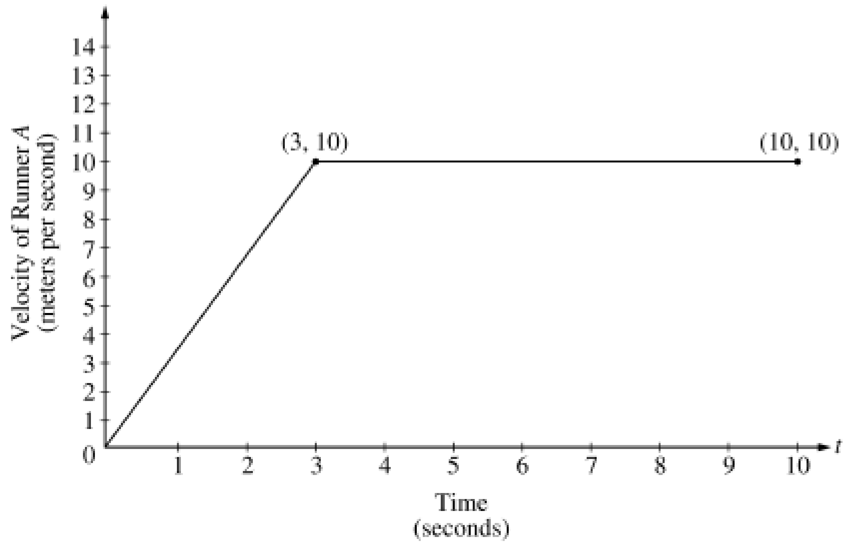
B. What is the position of the particle at time $t = 3$?

C. What is the average velocity from time $t = 0$ until time $t = 3$?

D. What is the average acceleration from time $t = 0$ until time $t = 3$?

LESSON 2A NOTES

3. AP:2000#2



2. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t + 3}$.

   (a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

LESSON 2A HW

1. A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^{t}\right)$. At time $t = 0$, the particle is at $y = -1$.

A. Find the total distance traveled by the particle between time $t = 0$ and time $t = 2$.

B. What is the position of the particle at time $t = 2$?

C. What is the average velocity over the time period $0 \leq t \leq 2$?

D. What is the average acceleration over the time period $0 \leq t \leq 2$?

LESSON 2A HW

2. AP:2011#1

1. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

   (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

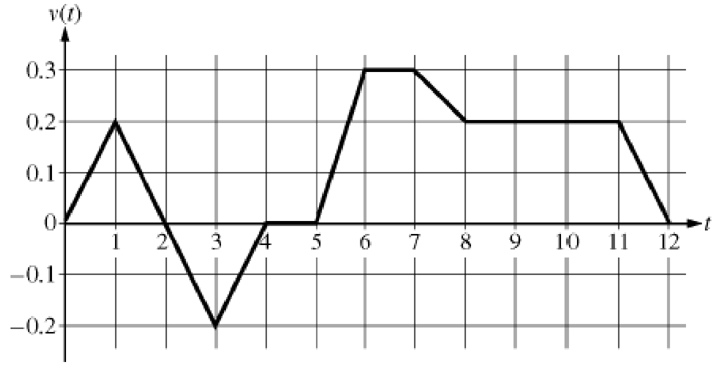
   (b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

   (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

   (d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

LESSON 2A HW

3. AP:2009#1



1. Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

   (a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

   (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

   (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

LESSON 2B APPLICATIONS
C2: Accumulating Quantity/(Rate In – Rate Out)

LESSON 2B NOTES

1. The rate at which people enter an amusement park on a given day is modeled by the function E defined by $E(t)=\dfrac{15600}{t^2-24t+160}$.

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by $L(t)=\dfrac{9890}{t^2-38t+370}$.

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours in which the park is open. At time $t = 9$, there are no people in the park.

A. How many people have entered the park by 5:00 pm ($t = 17$)? Round to the nearest whole number.
B. The price of admissions to the park is $15 until 5:00 pm. After 5:00 pm the price of admissions to the park is $11. How many dollars are collected in admissions to the park on the given day? Round to the nearest whole number.
C. Let $H(t)$ be the number of people in the park at time t. Find $H(17)$.
D. Write a function, involving an integral expression, for $H(t)$, the number of people in the park at time t.

LESSON 2B NOTES

2. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by $F(t) = 82 + 4\sin\left(\frac{t}{2}\right)$ for $0 \le t \le 30$, where F(t) is measured in cars per minute and t is measured in minutes.

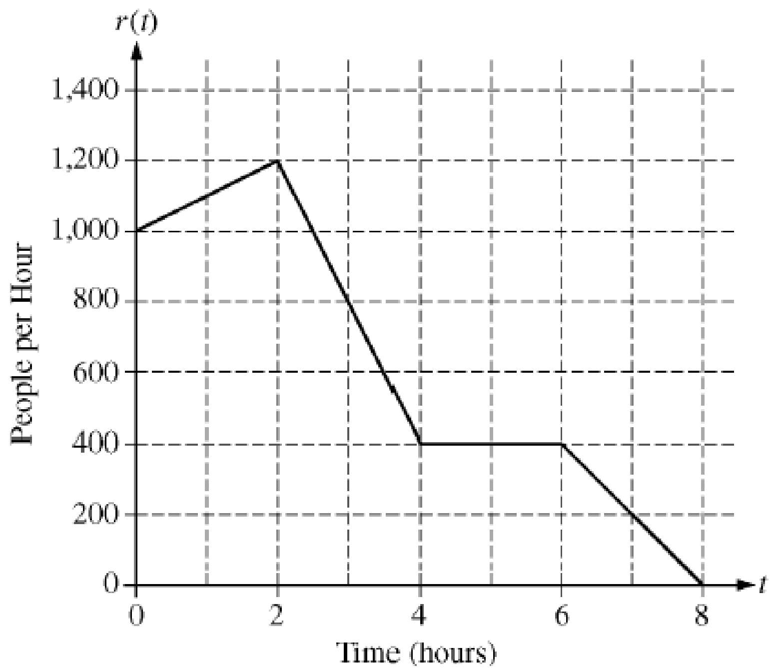
A. To the nearest whole number, how many cars pass through the intersection over the 30-minutes period?

B. What is the average traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

C. What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

LESSON 2B NOTES

3. AP:2010#3



3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

   (a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

   (b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

   (c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

   (d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

LESSON 2B HW

1. The tide removes sand from Sandy Point Beach at a rate modeled by the function *R*, given by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$.

A pumping station adds sand to the beach at a rate modeled by the function *S*, given by $S(t) = \frac{15t}{1+3t}$.

Both *R*(*t*) and *S*(*t*) have units of cubic yards per hour and *t* is measures in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

A. How much of the sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
B. Write a function, involving an expression, for *Y*(*t*), the total number of cubic yards of sand on the beach at time *t*.
C. Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.
D. Find the total number of cubic yards of sand on the bank at time $t = 4$.

LESSON 2B HW

2. AP:2003FB#2

2. A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

LESSON 2B HW

3. AP:2002FB#2

2. The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

   (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

   (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

   (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

   (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

LESSON 2B HW

4. AP:2010#1

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

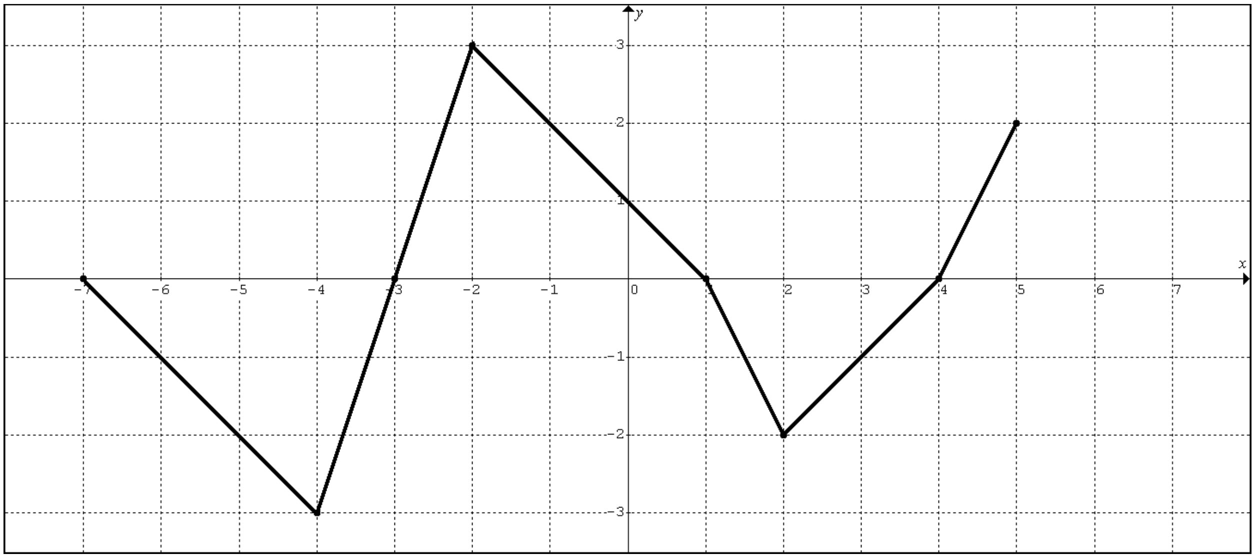
(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

BC: Q301: LESSON 3 – WARM UP

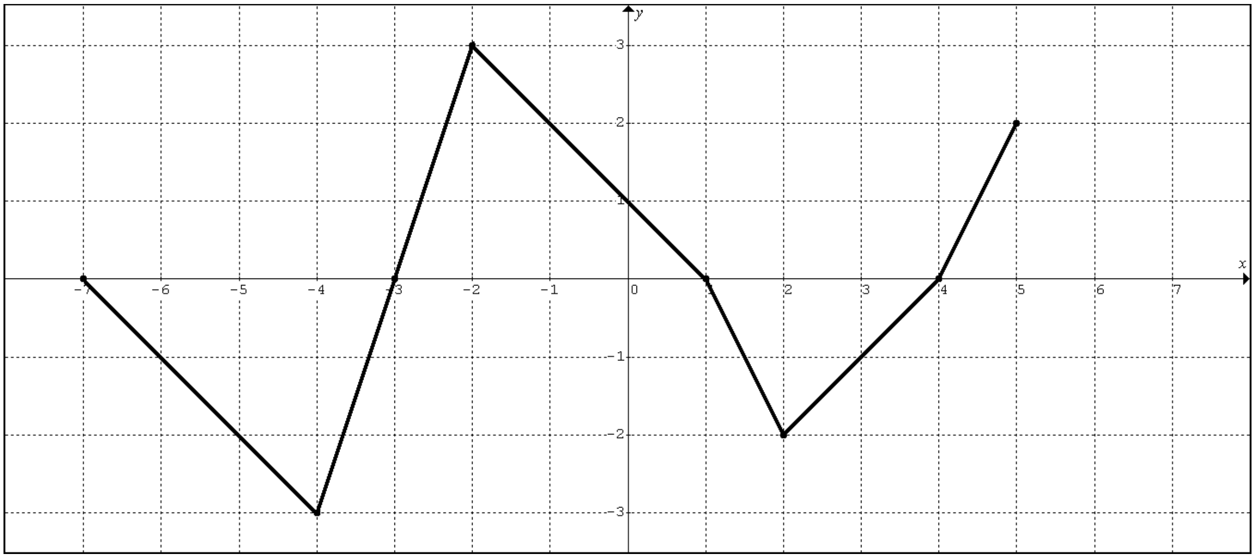


The graph above represents $f'(x)$ (the derivative of the function $f(x)$) on $-7 \le x \le 5$.
Suppose $f(-3) = 80$.

1. Evaluate $\int_{-7}^{5} f'(x)dx$

2. Find $f(5)$ and $f(-7)$

3. What is the average rate of change in $f(x)$ on the interval $-7 \le x \le 5$?



The graph above represents $f'(x)$ (the derivative of the function $f(x)$) on $-7 \le x \le 5$

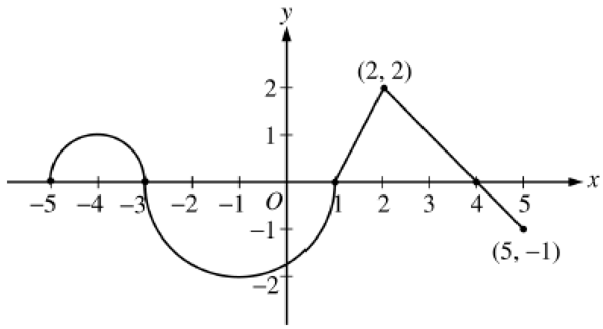
Suppose $f(-3) = 80$.

4. Find $f'(-1)$, $f''(-1)$, $f'(-4)$, and $f''(-4)$

5. For what value(s) of x will the function $f(x)$ have a local minimum? Justify.

6. For what value(s) of x will the function $f(x)$ be concave downward? Justify.

LESSON 3A: GUIDED PRACTICE 1: 2007FB #4



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

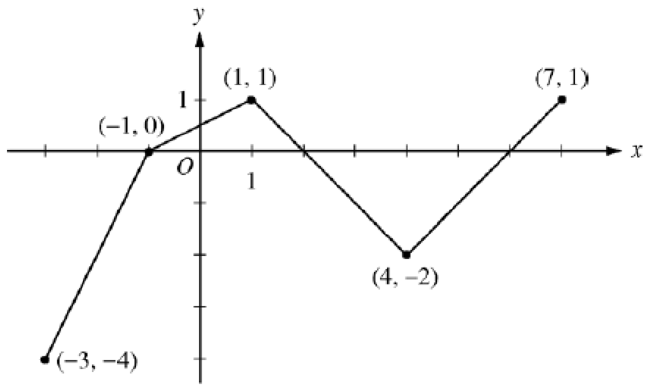
(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

LESSON 3A: PRACTICE 2: 2008FB#5



Graph of g'

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

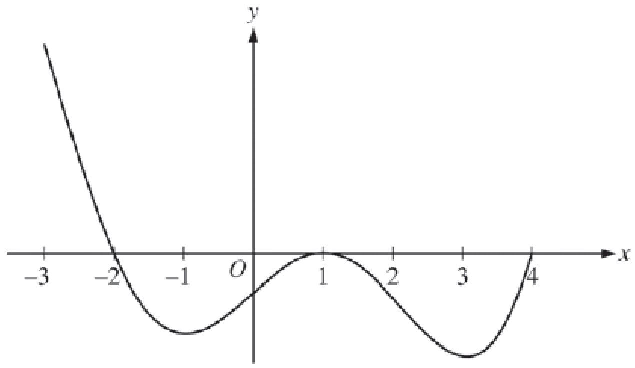
   (a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

   (b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

   (c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

   (d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

LESSON 3A: PRACTICE 3: 2015 #5



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

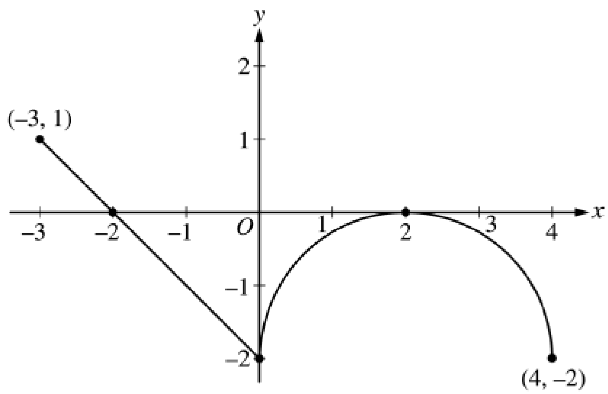
   (a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

   (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

   (c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

   (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

LESSON 3A: PRACTICE 4: 2003 #4



Graph of f'

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

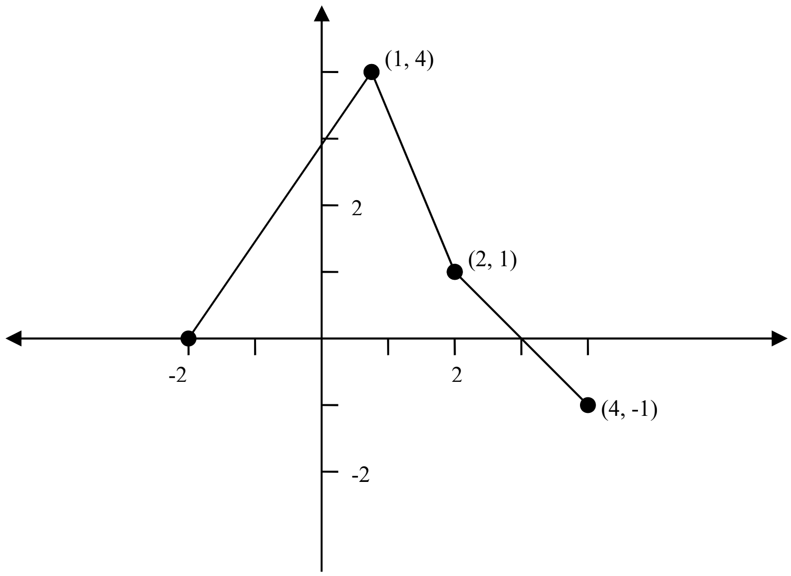
(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

LESSON 3B: THE FUNDAMENTAL THEOREM OF CALCULUS PART 1

Defining the relationship: $\int f(x)dx = g(x) + C$

Defining the relationship: $g(x) = \int_{a}^{x} f(t)dt$ (*a* is a constant)

LESSON 3B: GUIDED PRACTICE 1: 1999 #5 (adaptation)



EX1: The graph of the function *f*, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

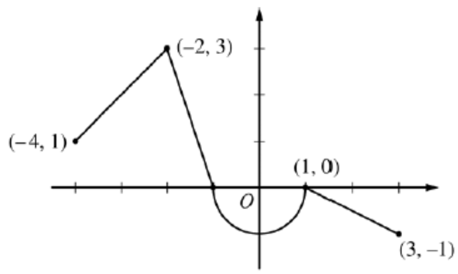
A. Compute $g(4)$ and $g(-2)$

B. Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

C. Find the absolute maximum and minimum values of g on the closed interval [-2, 4]. Justify your answer.

D. The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

LESSON 3B: PRACTICE 2: 2012 #3



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

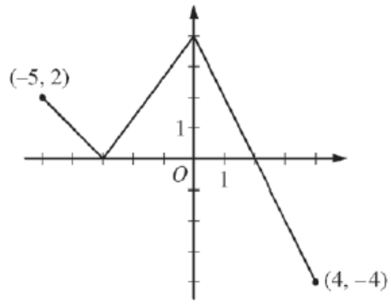
(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

LESSON 3B: PRACTICE 3: 2014 #3



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\, dt$.

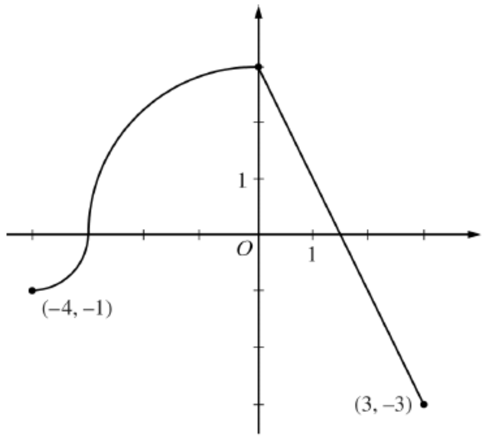
   (a) Find $g(3)$.

   (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

   (c) The function h is defined by $h(x) = \frac{g(x)}{5x}$. Find $h'(3)$.

   (d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

LESSON 3B: PRACTICE 4: 2011 #4



Graph of f

4. The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

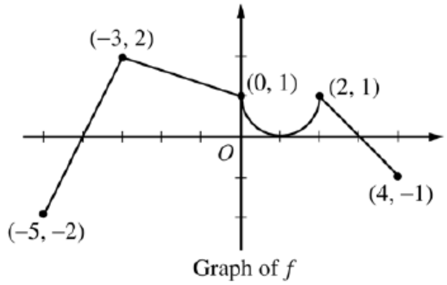
   (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

   (b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

   (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

   (d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

LESSON 3B: ADDITIONAL PRACTICE 1: 2004 #5



5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

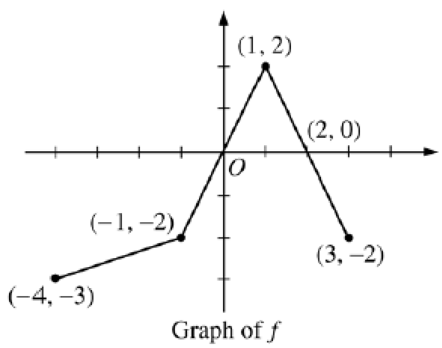
   (a) Find $g(0)$ and $g'(0)$.

   (b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

   (c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

   (d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

LESSON 3B: ADDITIONAL PRACTICE 2: *VERY CHALLENGING* 2005FB #5



4. The graph of the function f above consists of three line segments.

   (a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

   (b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

   (c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

   (d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

BC: Q301: LESSON 4 – APPROXIMATING A DEFINATE INTEGRAL

Approximating a Definite Integral with a Riemann Sum

RECTANGLE APPROXIMATION

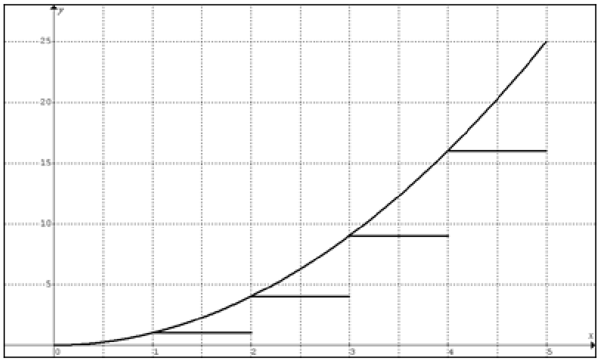
$\int_a^b f(x)dx \approx \sum_{k=1}^{n} f\left(c_k\right)\Delta x_k \; on\left[a,b\right]$ where $f(c_k)$ is the value of *f* at $x = c$ on the *k*th interval.

TRAPEZOIDAL APPROXIMATION

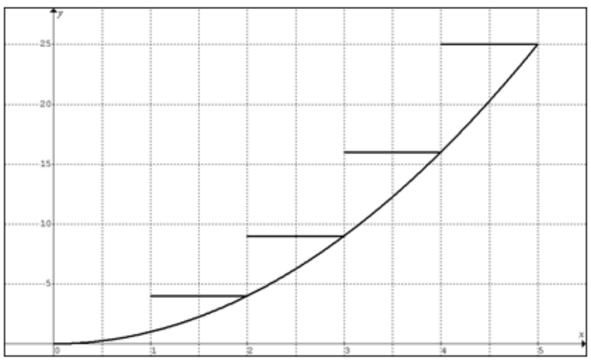
$\int_a^b f(x)dx \approx \frac{\Delta x}{2}\left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right)$ where $y = f(x)$ and Δx is constant.

1. Consider the area under the curve (bounded by the x-axis) of $f(x) = x^2$ from $x = 1$ to $x = 5$.

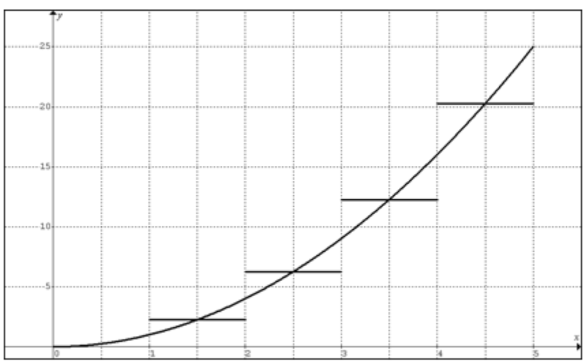
Use 4 equal rectangles whose heights are the left endpoint of each rectangle to approximate the area. (LRAM)



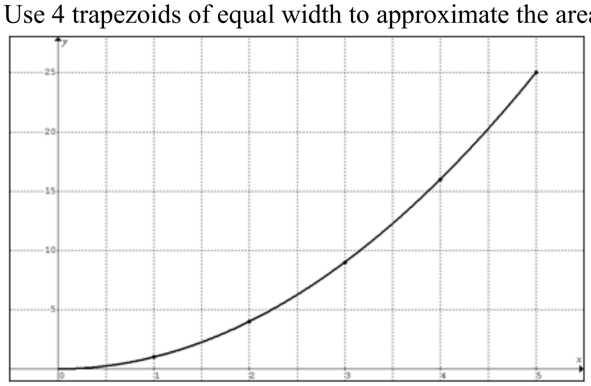
Use 4 equal rectangles whose heights are the right endpoint of each rectangle to approximate the area. (RRAM)



Use 4 equal rectangles whose heights are the midpoint of each rectangle to approximate the area. (MRAM)



Use 4 trapezoids of equal width to approximate the area. (TRAM)



2. Use LRAM, MRAM, and TRAM with $n = 3$ equal rectangles to estimate the $\int_{2}^{3.5} f(x)dx$ where values of the function $y = f(x)$ are as given in the table below.

x	2.0	2.25	2.5	2.75	3	3.25	3.5
y	3.2	2.7	4.1	3.8	3.5	4.6	5.2

3. Use a Trapezoidal approximation with $n = 3$ trapezoids to estimate the $\int_{2}^{5.5} f(x)dx$ where values of the function $y = f(x)$ are as given in the table below.

x	2.0	3	5	5.5
y	3.2	2.7	4.1	3.8

4.

t (hours)	$R(t)$ (gallons per hours)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above measured every 3 hours for a 24-hour period.

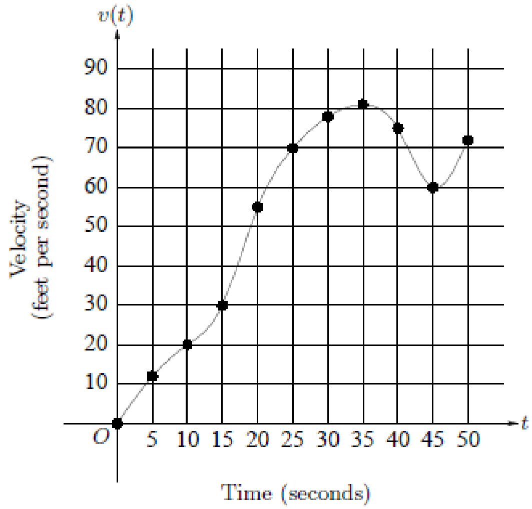
A. Using correct units, explain the meaning of the integral $\int_0^{24} R(t)dt$ in terms of water flow.

B. Use a trapezoidal approximation with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$.

C. Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate the average of $R(t)$.

D. The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the first 24-hour time period. Indicate the units of measure.

HW1: 1998 #3



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

HW2: 2004FB #3

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

   (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

   (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

   (c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.

   (d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

HW3: 2008 #2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

   (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

   (c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

   (d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

BC: Q301 LESSON 5: FTC-1 and THE CHAIN RULE

LESSON 5: BASIC PRACTICE Find $\frac{dy}{dx}$ for each equation below.

1. $y = \int_{7}^{x} \frac{1+t}{1+t^2}\,dt$

2. $y = \int_{0}^{x^2} e^{t^2}\,dt$

3. $y = \int_{x}^{6} \ln\left(1+t^2\right)dt$

4. $y = \int_{x^3}^{5} \frac{\cos t}{t^2+2}\,dt$

BC: Q301 LESSON 5 REVIEW PRACTICE EXERCISE

x	-2	0	1	5	7	8
$f(x)$	14	5	2	-3	-10	9

Let $g(x)=\int_0^x f(t)dt$ where $f(x)$ is a differentiable function with specific values as shown above.

1. Estimate $g(8)$ using a trapezoidal approximation with four trapezoids.

2. Find $g'(1)$

3. Estimate $g''(1.3)$

4. If $h(x)=\int_5^x f(t)dt$, estimate $h(-2)$ using a left Riemann sum with three rectangles.

5. If $w(x)=\int_{-2}^{3x^2+2} f(t)dt$, find $w'(1)$.

6. If $n(x)=\int_x^0 f(t)dt$, find $n'(1)$.

BC. Q301 LESSON5 (MVT REVIEW PRACTICE) – CALCULATORS REQUIRED

Let $f(x) = \ln(x+2) + \sin(x)$ on $[-1, 4.5]$.

1. Find the average rate of change in *f* on [-1, 4.5].

2. Find the value(s) of *x* that satisfy the conclusion to the Mean Value Theorem for derivatives.

3. Find the average value of *f* on [-1, 4.5].

4. Find the value(s) of *x* that satisfy the conclusion to the Mean Value Theorem for integrals.