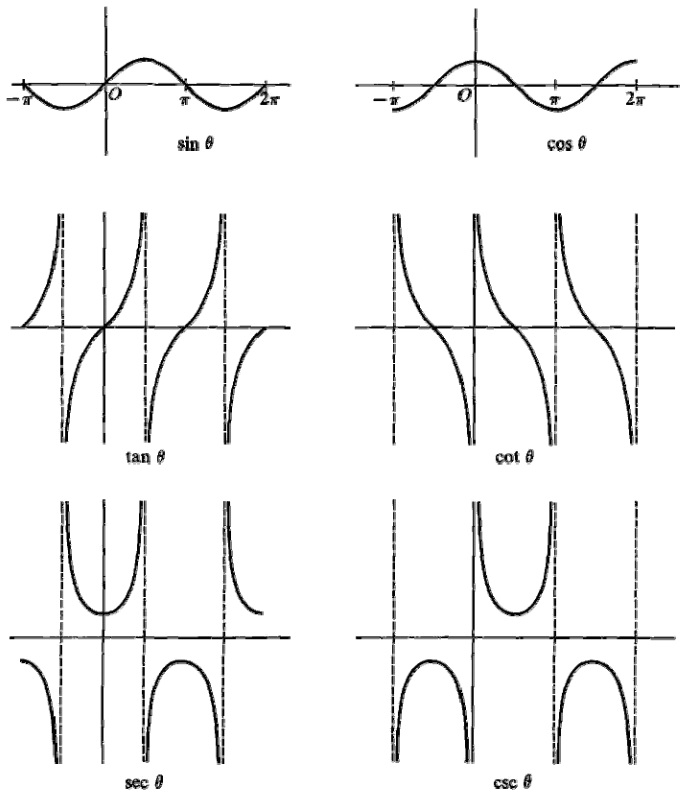
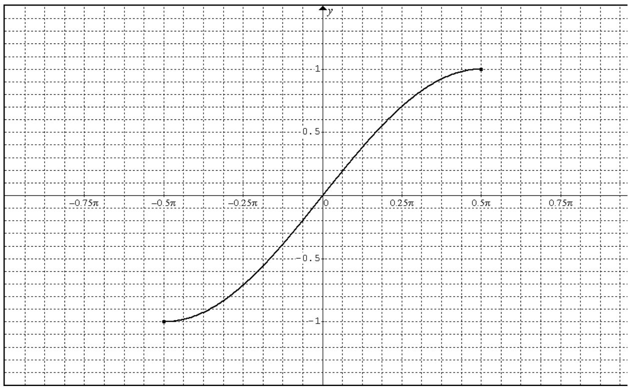
BC.Q104.NOTES: Chapter 3.8, 3.9 – Lesson 1

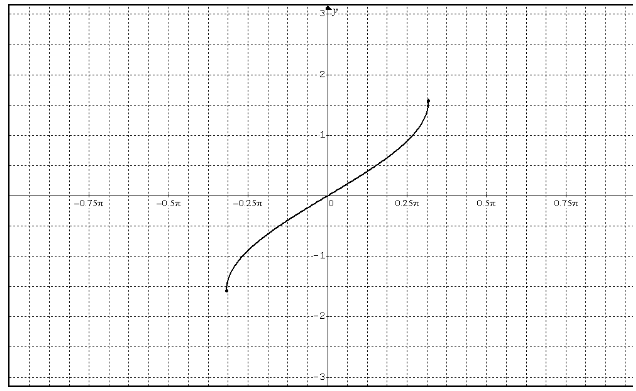
Quick Reference



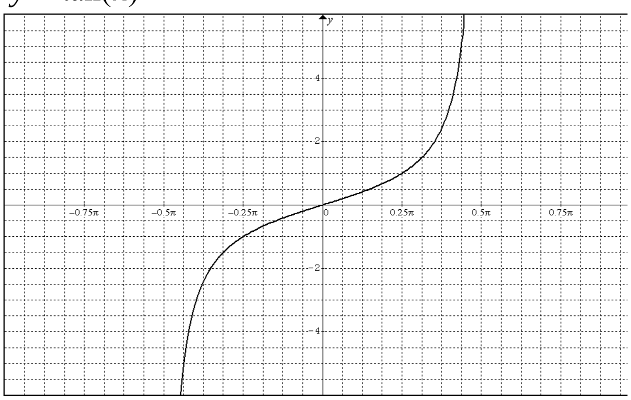
$y = \sin(x)$



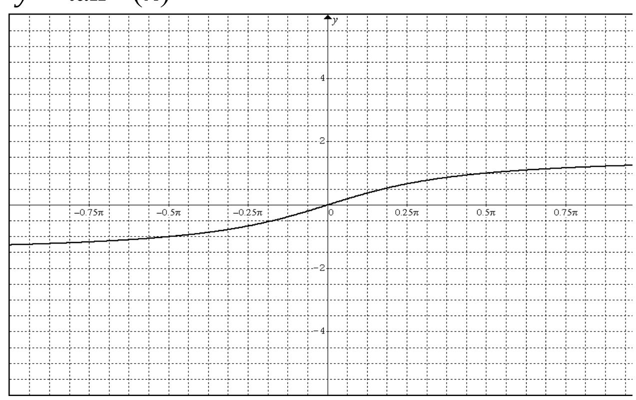
$y = \sin^{-1}(x)$



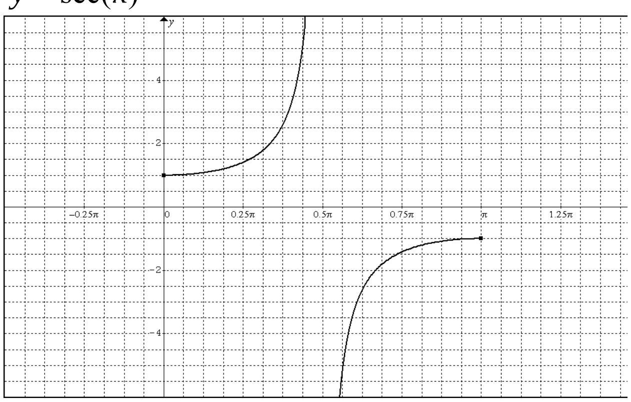
$y = \tan(x)$



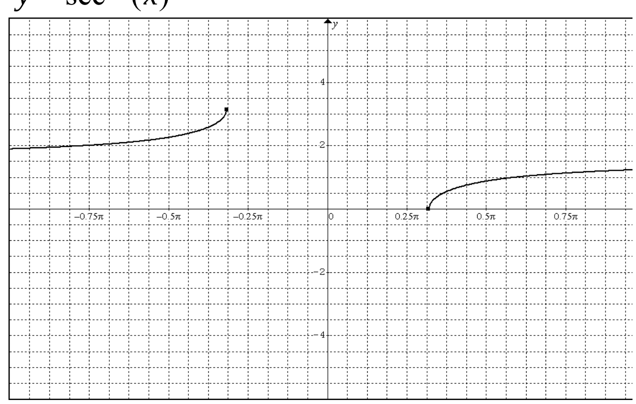
$y = \tan^{-1}(x)$



$y = \sec(x)$



$y = \sec^{-1}(x)$



Development of Inverse Trigonometric Function Derivatives

Development of Transcendental Function Derivatives

Practice Problems

Practice Problems Continued

LESSON 1 HW

Section 3.8: #1 – 21 odd, 35-40 M.C.

Section 3.9: #5-25 odd, 31, 49

(LEAVE THE ANSWERS UNSIMPLIFIED)

SECTION 3.8	SECTION 3.9
1. $y=\cos^{-1}\left(x^2\right)$ Find $\frac{dy}{dx}$	5. $y=e^{2x/3}$ Find $\frac{dy}{dx}$
3. $y=\sin^{-1}\sqrt{2}t$ Find $\frac{dy}{dt}$	**7.** $y=e^2x-e^x$ Find $\frac{dy}{dx}$
5. $y=\sin^{-1}\frac{3}{t^2}$ Find $\frac{dy}{dt}$	9. $y=e^{\sqrt{x}}$ Find $\frac{dy}{dx}$
7. $y=x\sin^{-1}x+\sqrt{1-x^2}$ Find $\frac{dy}{dx}$	11. $y=8^x$ Find $\frac{dy}{dx}$
9. $x=\sin^{-1}\left(\frac{t}{4}\right)$ Find $x'(3)$	13. $y=3^{\csc x}$ Find $\frac{dy}{dx}$
11. $x=\tan^{-1}t$ Find $x'(2)$	15. $y=\ln\left(x^2\right)$ Find $\frac{dy}{dx}$
13. $y=\sec^{-1}(2s+1)$ Find $\frac{dy}{ds}$	17. $y=\ln\left(\frac{1}{x}\right)$ Find $\frac{dy}{dx}$
15. $y=\csc^{-1}\left(x^2+1\right),\ x>0$ Find $\frac{dy}{dx}$	19. $y=\ln(\ln x)$ Find $\frac{dy}{dx}$
17. $y=\sec^{-1}\frac{1}{t},\quad 0<t<1$ Find $\frac{dy}{dt}$	21. $y=\log_4 x^2$ Find $\frac{dy}{dx}$
19. $y=\cot^{-1}\sqrt{t-1}$ Find $\frac{dy}{dt}$	23. $y=\log_2\left(\frac{1}{x}\right)$ Find $\frac{dy}{dx}$
21. $y=\tan^{-1}\sqrt{x^2-1}+\csc^{-1}x,\quad x>1$ Find $\frac{dy}{dx}$	25. $y=\ln 2\cdot\log_2 x$ Find $\frac{dy}{dx}$

3.9 #31

A line with slope m passes through the origin and is tangent to $y = \ln(2x)$.

What is the value of m?

3.9 #49

Find an equation for the line tangent to the graph of $y = e^x$ and goes through the origin.

3.8 #35-40 Multiple Choice

35. **True or False** The domain of $y = \sin^{-1}x$ is $-1 \le x \le 1$. Justify your answer.

36. **True or False** The domain of $y = \tan^{-1}x$ is $-1 \le x \le 1$. Justify your answer.

37. **Multiple Choice** Which of the following is $\frac{d}{dx}\sin^{-1}\left(\frac{x}{2}\right)$?

(A) $-\frac{2}{\sqrt{4-x^2}}$ (B) $-\frac{1}{\sqrt{4-x^2}}$ (C) $\frac{2}{4+x^2}$

(D) $\frac{2}{\sqrt{4-x^2}}$ (E) $\frac{1}{\sqrt{4-x^2}}$

38. **Multiple Choice** Which of the following is $\frac{d}{dx}\tan^{-1}(3x)$?

(A) $-\frac{3}{1+9x^2}$ (B) $-\frac{1}{1+9x^2}$ (C) $\frac{1}{1+9x^2}$

(D) $\frac{3}{1+9x^2}$ (E) $\frac{3}{\sqrt{1-9x^2}}$

39. **Multiple Choice** Which of the following is $\frac{d}{dx}\sec^{-1}(x^2)$?

(A) $\frac{2}{x\sqrt{x^4-1}}$ (B) $\frac{2}{x\sqrt{x^2-1}}$ (C) $\frac{2}{x\sqrt{1-x^4}}$

(D) $\frac{2}{x\sqrt{1-x^2}}$ (E) $\frac{2x}{\sqrt{1-x^4}}$

40. **Multiple Choice** Which of the following is the slope of the tangent line to $y = \tan^{-1}(2x)$ at $x = 1$?

(A) $-2/5$ (B) $1/5$ (C) $2/5$ (D) $5/2$ (E) 5

LESSON 1 HW EXTENSION

1. IN Section 3.8 #5 $\frac{dy}{dt}=\frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}}\cdot -6t^{-3}$ This answer simplifies to $\frac{-6}{t\sqrt{t^4-9}}$.

SHOW HOW.

2. IN Section 3.8 #17 $\frac{dy}{dt}=\frac{1}{\left|\frac{1}{t}\right|\sqrt{\left(\frac{1}{t}\right)^2-1}}\cdot\frac{-1}{t^2}$ This answer simplifies to $\frac{-1}{\sqrt{1-t^2}}$.

SHOW HOW.

3. IN Section 3.8 #37 $\frac{dy}{dx}=\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\cdot\frac{1}{2}$ This answer simplifies to $\frac{1}{\sqrt{4-x^2}}$.

SHOW HOW.

BC.Q104.NOTES: Chapter 3.8, 3.9 – Lesson 2

PART I. Derivatives with Log Properties and Logarithmic Differentiation

<u>First Using Log Properties to Find a Derivative</u>

Logarithmic Differentiation

Part II. Calculus of General Inverses Functions

THM: If the domain of a function *f* is an interval on which $f'(x) > 0 \quad \forall x$ or on which $f'(x) < 0 \quad \forall x$, then the inverse of *f* is also a function.

$f'(x) > 0$ implies that *f* is increasing.
$f'(x) < 0$ implies that *f* is decreasing.

FORMULA:

Example: Consider $f(x) = x^5 + x + 1$ on $(-\infty, \infty)$.

1. Prove that the inverse of $f(x)$ is also a function?

2. Find the slope of the inverse function f^{-1} at $x = 3$.

EXAMPLE 2: $f(x) = 2x^3 + 5x + 3$ is a one-to-one function.

EXAMPLE 3: $f(x) = 5x^3 + x - 7$ is a one-to-one function.

EXAMPLE 4: $f(x) = 2x^5 + x^3 + 1$ is a one-to-one function. Let $g(x) = f^{-1}(x)$

Formula Development

Task: Construct a relationship between the slope of a function *f* at (a, b) and the slope of the inverse function at (b, a).

LESSON 2 HW

Section 3.9 #43, 45, 46, and 47 + MC 2 and 3
Section 3.8 #28, 29 + extra problem

3.9 #43: $y = (\sin x)^x$, $0 < x < \pi/2$. Find $\dfrac{dy}{dx}$ using logarithmic differentiation.

3.9 #45: $y = \sqrt[5]{\dfrac{(x-3)^4(x^2+1)}{(2x+5)^3}}$. Find $\dfrac{dy}{dx}$ using logarithmic differentiation.

3.9 #45: $y = \dfrac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$. Find $\dfrac{dy}{dx}$ using logarithmic differentiation.

3.9 #47: $y = x^{\ln x}$. Find $\dfrac{dy}{dx}$ using logarithmic differentiation.

2. Multiple Choice Which of the following gives dy/dx if $y = \cos^3(3x-2)$?

(A) $-9\cos^2(3x-2)\sin(3x-2)$

(B) $-3\cos^2(3x-2)\sin(3x-2)$

(C) $9\cos^2(3x-2)\sin(3x-2)$

(D) $-9\cos^2(3x-2)$

(E) $-3\cos^2(3x-2)$

3. Multiple Choice Which of the following gives dy/dx if $y = \sin^{-1}(2x)$?

(A) $-\dfrac{2}{\sqrt{1-4x^2}}$ (B) $-\dfrac{1}{\sqrt{1-4x^2}}$ (C) $\dfrac{2}{\sqrt{1-4x^2}}$

(D) $\dfrac{1}{\sqrt{1-4x^2}}$ (E) $\dfrac{2x}{1+4x^2}$

3.8 #28: Let $f(x) = x^5 + 2x^3 + x - 1$

(A) Prove that the inverse of *f* is also a function

(B) Find $f(1)$ and $f'(1)$

(C) Find $f^{-1}(3)$ and $\left(f^{-1}\right)'(3)$

3.8 #29: Let $f(x) = \cos x + 3x$. Also let $g(x) = f^{-1}(x)$

(A) Prove that the inverse of *f* is also a function

(B) Find $g'(1)$

EXTRA PROBLEM: Find a positive value of $x = a$ such that the tangent to $f(x) = x^2 + 4$ at $x = a$ also passes through the point (0,2).