Q103.BC.Notes: Chapter 3.7, 4.6, 4.5

LESSON 1 – 3.7 Implicit Differentiation

INTRODUCTION:

Identify the Shape: $x^2 + y^2 = 1$, $9x^2 + 4y^2 = 40$, $2x^2 - y^2 = 3$, $y^4 + 3y - 4x^3 = 5x + 1$

What does $\frac{dy}{dx}$ represent in reference to the equations above?

Example 1: $y^4 + 3y - 4x^3 = 5x + 1$

A. Find $\frac{dy}{dx}$.

B. Find the slope of the tangent line at the point P(1, -2)

C. Use implicit differentiation to find $\frac{d^2y}{dx^2}$.

2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if $x^2 + y^2 = 1$

3. Find $\frac{dy}{dx}$ if $4xy^3 - x^2y + x^3 - 5x + 6 = 0$

4. Find $\frac{dy}{dx}$ if $y = x^2 \sin y$

5. Find $\frac{dy}{dx}$ if $\tan(xy) + x^2 y + 2 = x$.

Find an equation of the line tangent to the curve at the point when $y = 0$.

AP Question

Consider the curve given by $xy^2 - x^3 y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2 y - y^2}{2xy - x^3}$

(b) Find all the points on the curve whose *x*-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the *x*-coordinate of each point on the curve where the tangent line is vertical.

LESSON 1 HW:
AP QUESTION ON PREVIOUS PAGE + SECTION 3.7: # 1, 5, 17, 21, 49, 56, 57

(*Section 3.7*)

#1: Let $x^2y + xy^2 = 6$ Find $\frac{dy}{dx}$.

#5: Let $x = \tan(y)$ Find $\frac{dy}{dx}$.

#17: Let $x^2 + xy - y^2 = 1$
Find an equation of both the tangent and normal lines to the curve at the point (2, 3).

#21: Let $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$
Find an equation of both the tangent and normal lines to the curve at the point (-1, 0).

#29: Let $y^2 = x^2 + 2x$ Find $\frac{dy}{dx}$ and then $\frac{d^2y}{dx^2}$.

#49: Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

#56: The line that is normal to the curve $x^2 + 2xy - 3y^2 = 0$ at (1, 1) intersects the curve at what other point?

#57 Find the normals to the curve $xy + 2x - y = 0$ that are parallel to the line $2x + y = 0$

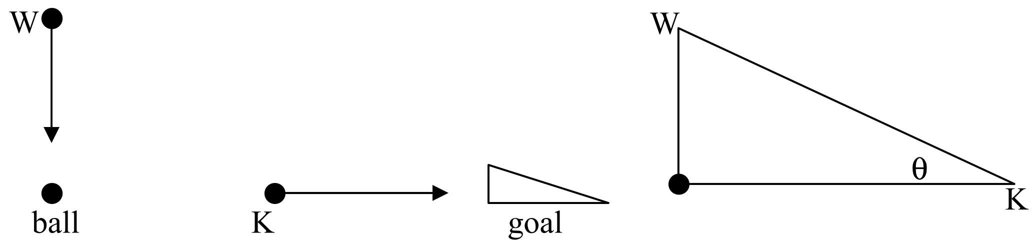
Lesson 2 - Chapter 4.6 Related Rates

Ex: 1 A ladder 20 ft long leans against the wall of a vertical building. If the ladder slides away from the building horizontally at a rate 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 12 ft above the ground?

Ex: 2 A camera is mounted at a point 3000 ft from the base of a rocket launching pad. If the rocket is rising vertically at 880 ft/s when it is 4000 ft above the launching pad, how fast must the camera elevation angle change at that instant to keep the camera aimed at the rocket?

Ex: 3 A water tank has the shape of an inverted right circular cone of altitude 12 ft and base 6 ft. If water is being pumped into the tank at a rate of 1.2 ft^3/min, approximate the rate at which the water level is rising when the water is 3 ft deep.

EX 4: A urinal is 15 ft long and 4 ft across the top. Its ends are isosceles triangles with height 3 ft. Pee runs into the urinal at the rate of 2.5 ft^3/min. How fast is the pee rising when it is 2 ft deep?



EX 5: Walter approaches the soccer ball, due south, at a rate of 12 ft/s as Kevin approaches the soccer goal, due east, at 10 ft/s. When Walter is 6 ft from the ball and Kevin is 8 ft from the ball …

(a) What is the change in rate of the distance between the two boys?

(b) What is the change in rate of angle θ?

LESSON 2 HW:
SECTION 4.6 #11, 21, 29, 34, 17 EXTRAS: 4.6 # 14, 16, 19, 25

11. *Inflating Balloon* A spherical balloon is inflated with helium at the rate of 100π ft^3/min.

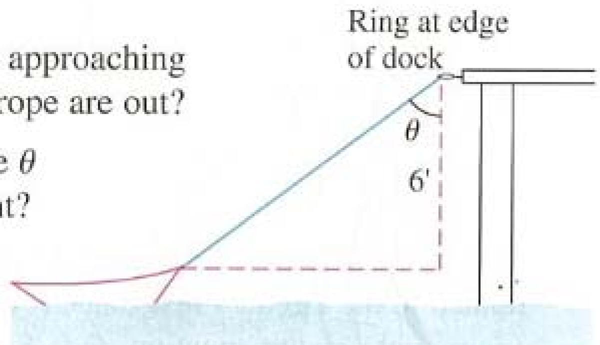
(a) How fast is the balloon's radius increasing at the instant the radius is 5 ft?

(b) How fast is the surface area increasing at that instant?

21. *Hauling in a Dinghy* A dinghy is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow as shown in the figure. The rope is hauled in at the rate of 2 ft/sec.

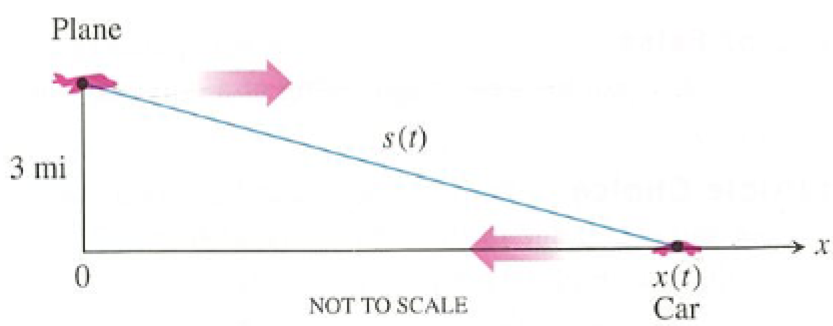
(a) How fast is the boat approaching the dock when 10 ft of rope are out?

(b) At what rate is angle θ changing at that moment?

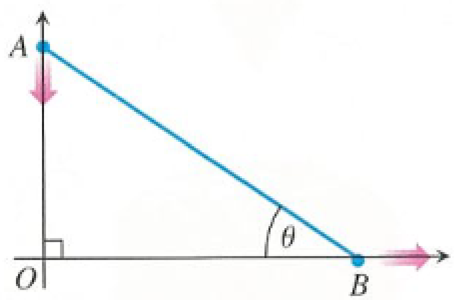


29. *Moving Shadow* A man 6 ft tall walks at the rate of 5 ft/sec toward a streetlight that is 16 ft above the ground. At what rate is the length of his shadow changing when he is 10 ft from the base of the light?

32. *Speed Trap* A highway patrol airplane flies 3 mi above a level, straight road at a constant rate of 120 mph. The pilot sees an oncoming car and with radar determines that at the instant the line-of-sight distance from plane to car is 5 mi the line-of-sight distance is decreasing at the rate of 160 mph. Find the car's speed along the highway.



34. *Walkers* A and B are walking on straight streets that meet at right angles. A approaches the intersection at 2 m/sec and B moves away from the intersection at 1 m/sec as shown in the figure. At what rate is the angle θ changing when A is 10 m from the intersection and B is 20 m from the intersection? Express your answer in degrees per second to the nearest degree.

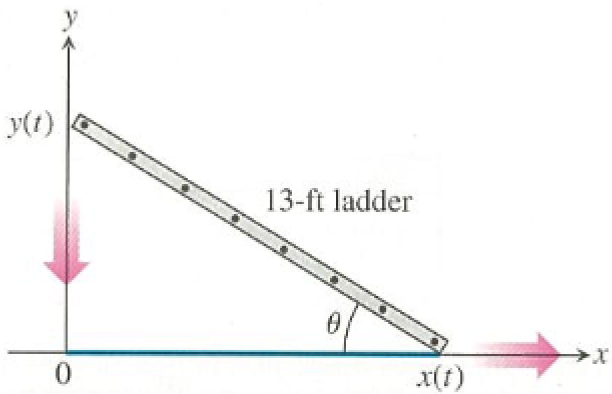


17. *Draining Conical Reservoir* Water is flowing at the rate of 50 m^3/min from a concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. **(a)** How fast is the water level falling when the water is 5 m deep? **(b)** How fast is the radius of the water's surface changing at that moment? Give your answer in cm/min.

14. *Flying a Kite* Inge flies a kite at a height of 300 ft, the wind carrying the kite horizontally away at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

16. *Growing Sand Pile* Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the **(a)** height and **(b)** radius changing when the pile is 4 m high? Give your answer in cm/min.

19. ***Sliding Ladder*** A 13-ft ladder is leaning against a house (see figure) when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.



(a) How fast is the top of the ladder sliding down the wall at that moment?

(b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing at that moment?

(c) At what rate is the angle θ between the ladder and the ground changing at that moment?

25. ***Particle Motion*** A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (in meters) increases at a constant rate of 10 m/sec. How fast is the angle of inclination θ of theline joining the particle to the origin changing when $x = 3$?

Lesson 3 – Chapter 4.5 Notes (Linear Approximation)

GRAPHICAL REPRESENTATION

1. Find the linearization of (line tangent to) $f(x) = 2x^2 - 3x$ at $x = 2$.

2. Use the linearization (tangent line) to approximate $f(2.5)$.

3. Estimate the change in $f(x) = 3x - x^4$ when x increases from 1 to 1.05.

Lesson 3 – Practice

1. Let $f(x)=\sqrt{x^2+9}$.
Use a linearization of *f* at $x=-4$ and use it to approximate $f(-3.9)$.

2. Let $f(x)=\ln(x+1)$.
Use a linearization of *f* at $x=0$ and use it to approximate $f(0.2)$.

3. Let *f* be a function with $f(0)=2$ and $f'(x)=e^x\cos(x)$.
Use a linearization of *f* at $x=0$ and use it to approximate $f(0.5)$

4. Let *f* be a function with $f(0)=10.2$ and $f'(x)=2\sin\left(\frac{\pi}{2}-x\right)$.
Use a linearization of *f* at $x=0$ and use it to approximate $f(-0.3)$

5. Use a linearization to estimate $\sqrt{1.01}$

6. Estimate the change in $f(x) = x^3 + 2x$ when x increases from 1 to 1.03.

7. Estimate the change in volume of a sphere if the radius goes from 10 to 10.05 cm.

8. Estimate the change in area of a circle if the radius goes from 5 to 5.2 mm.

Lesson 3 (SECTION 4.5) – Homework

1. Let $f(x) = 2 + \sin x$.
Use a linearization of *f* at $x = 0$ and use it to approximate $f(-0.1)$.

2. Let $f(x) = x^3 - 2x + 3$.
Use a linearization of *f* at $x = 2$ and use it to approximate $f(1.8)$.

3. Let *f* be a function with $f(3) = 7$ and $f'(x) = \sqrt{1+x}$.
Use a linearization of *f* at $x = 3$ and use it to approximate $f(2.5)$.

4. Let *f* be a function with $f(0) = 4$ and $f'(x) = \cos(x^2)$.
Use a linearization of *f* at $x = 0$ and use it to approximate $f(0.4)$.

5. Let *f* be a function with $f(4) = 3$ and $f'(x) = 2 - x$.
Use a linearization of *f* at $x = 4$ and use it to approximate $f(4.2)$.

6. Estimate the change in $f(x) = x^2 - 5x$ as *x* increase from 2 to 2.1

7. Estimate the change in volume of a cube if edge decreases from 9 to 8.99 mm.

8. Estimate the change in circumference of a circle if the diameter changes from 2 to 2.02 in.

9. Use a linearization to approximate $\sqrt[3]{26}$.

10. AP Related Rates Problems: 1987 AB5 and 1990 AB4

11. EXTRA related rates problems from Lesson 2 HW

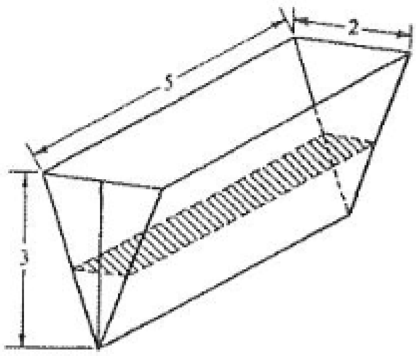
1987 – AB5

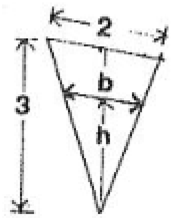
5. The trough shown in the figure above is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

   (a) Find the volume of water in the trough when it is full.

   (b) What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

   (c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?





1990 - AB 4

4. The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second.
(Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

(b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

(c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?