Q102.BC.NOTES: Chapter 3.3, 3.5, 3.6

BC.Q102.LESSON 1 (3.3)

3.3 Techniques of Differentiation

Let $y = f(x)$.
Notation for the derivative:
Lagrange: $f'(x)$
Leibniz: $\frac{dy}{dx}$
Newton: $\dot{y}$

Rules for the derivative

1. $\frac{d}{dx}[c] = 0$

2. $\frac{d}{dx}[x^n] = nx^{n-1}$ (THE POWER RULE)

3. $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$

4. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

5. $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$ (THE PRODUCT RULE)

6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$ (THE QUOTIENT RULE)

LESSON 1: Examples

LESSON 1: Continued

1. Find the first three derivatives of the function. Use Lagrange and Leibniz notation.

2. (Theory – Notational Examples) Express each derivative using the appropriate properties.

3. Prove the Power Rule:

4. Prove the Product Rule:

3.3 HW

1. $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + x$; Find $f'(x)$

2. $y = 1 - x + x^2 - x^3 + 5\sqrt{x} - \frac{6}{x^5}$; Find $\frac{dy}{dx}$

3. $g(x) = \frac{2x+5}{3x-2}$; Find $g'(x)$

4. $y = \frac{5x^3 - 2x + \frac{1}{x}}{7x^5 - 2x}$; Find $\frac{dy}{dx}$

5. $h(x) = (x^2+1)(x^3+5x-4)$; Find $h'(x)$

6. $y = (3x^{-2} + 5x^2 - x)(\sqrt{x} + 2x - 1)$; Find $\frac{dy}{dx}$

7. $f(x) = \frac{2\sqrt{x} - \frac{1}{\sqrt{x}} + 3}{4x^{-2} - 8x}$; Find $f'(x)$

8. Find the value(s) of x where the tangents to $y = x^3 - 2x^2 + x + 1$ are horizontal

9. USE Lagrange and Leibniz notation to find the first four derivatives of $y = x^4 + x^3 - 2x^2 + x - 5$

10. Find the equation of the tangent lines to $y = x^3 + x$ where the slope is 4.

11. Find the points on $y = 2x^3 - 3x^2 - 12x + 20$ where the tangent is parallel to the x-axis.

12. Find the equation of the line tangent to $y = x^3$ at $x = -2$.

13. Find the equation of the line tangent to $y = \frac{4x}{x^2+1}$ at $x = 0$ and at $x = 1$.

BC.Q102.**Lesson 2 (3.5)**

3.5: Derivatives of Trigonometric Functions

Important Limits: $\lim_{h\to 0} \frac{\sin h}{h}$ and $\lim_{h\to 0} \frac{1-\cos h}{h}$

Important Identities: $\sin(x+y)$ $\cos(x+y)$

1. Use the definition of derivative to find $\frac{d}{dx}[\sin x]$

2. Use the definition of derivative to find $\frac{d}{dx}[\cos x]$

3. Find $\frac{d}{dx}[\tan x]$

4. Find $\frac{d}{dx}[\sec x]$

~Book of Memories~
(Entry 1)

$$\frac{d}{dx}[\sin x] =$$

$$\frac{d}{dx}[\cos x] =$$

$$\frac{d}{dx}[\tan x] =$$

$$\frac{d}{dx}[\cot x] =$$

$$\frac{d}{dx}[\sec x] =$$

$$\frac{d}{dx}[\csc x] =$$

LESSON 2: Examples

Ex: 1 Find $f^{/}(x)$ if $f(x) = x^2 \tan x$

Ex: 2 Find $\dfrac{dy}{dx}$ if $y = \dfrac{\sin x}{1+\cos x}$

Ex: 3 Find $y^{//}\left(\dfrac{\pi}{4}\right)$ if $y = \sec x$

LESSON 2: Examples Continued

3.5 HW

1. $y = 1 + x - \cos x$; find $\frac{dy}{dx}$

2. $y = \frac{1}{x} + 5 \sin x$; find $\frac{dy}{dx}$

3. $y = 4 - x^2 \sin x$; find $\frac{dy}{dx}$

4. $y = \frac{4}{\cos x}$; find $\frac{dy}{dx}$

5. $y = \frac{\cot x}{1 + \cot x}$; find $\frac{dy}{dx}$

6. Find the equations for the lines that are tangent and normal to the graph of $y = \sin x + 3$ at $x = \pi$.

7. Find the equations for the lines that are tangent and normal to the graph of $f(x) = x^2 \sin x$ at $x = 3$.

8. Find the equations for the lines that are tangent and normal to the graph of $y = \sqrt{2} \cos x$ at $x = \frac{\pi}{4}$.

9. Find the equation of the horizontal tangent to $y = 4 + \cot x - 2 \csc x$

10. Find $\frac{d^2y}{dx^2}$ for $y = \csc x$

11. Find $\frac{d^{725}}{dx^{725}}(\sin x)$

Additional HW Problems

1. Find $\frac{d^{87}(\sin x)}{dx^{87}}$ (the 87th derivative of $\sin x$)

2. Let $f(x) = \cos x$. Find all positive integers n for which $f^{n}(x) = \sin x$.

3. Without using any trigonometric identity, find $\lim_{x\to 0} \frac{\tan(x+y)-\tan(y)}{x}$.

4.Let $y = 3 + 2\sin x$.

(a) Find the x-coordinate of all points on the graph at which the tangent line is parallel to the line $y = \sqrt{2}x - 5$

(b) Find an equation of the tangent line to the graph at the point on the graph with x-coordinate $\pi/6$.

BC.Q102.Lesson 3 (3.6)

Chapter 3.6 The Chain Rule

COMPOSITE FUNCTIONS REVIEW:

THE CHAIN RULE:

If $y = f(u)$, $u = g(x)$, and the derivatives $\frac{dy}{du}$ and $\frac{du}{dx}$ both exist, then the composite function defined by $y = f(g(x))$ has a derivative given by $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = f'(u)g'(x) = f'(g(x))g'(x)$.

LESSON 3: Examples

LESSON 3: Examples Continued

LESSON 3: Theory – Notational Examples

BC.Q102: LESSON 3 – NOTATIONAL EXAMPLES AND CHART EVALUATIONS

#1.

x	f	f'	g	g'	h	h'
0	2	11	-2	7	6	5
1	-4	5	5	8	-1/2	3
2	1	3	6	-1	0	4
6	-10	1/2	3	10	3/2	0

A. Find $\left[f(x)+3g(x)\right]'$ at $x=2$.

B. Find $\left[f(g(x))\right]'$ at $x=2$.

C. Find $\left[f(2x-g(h(x))\right]'$ at $x=2$.

#2. "CHART PRACTICE"

x	f	f'	g	g'
2	8	1/3	2	-3
3	3	2π	-4	5

A. Find $[f(g(x))]'$ at $x = 2$

B. Find $\left[\sqrt{f(x)}\right]'$ at $x = 2$

C. Find $\left[\frac{1}{g^2(x)}\right]'$ at $x = 3$

D. Find $\left[\sqrt{f^2(x) + g^2(x)}\right]'$ at $x = 2$

#3. “CHART PRATICE”

x	f	f'	g	g'
0	1	5	1	1/3
1	3	-1/3	-4	-8/3

A. Find $\left[f(g(x))\right]'$ at $x = 0$

B. Find $\left[(g(x) + f(x))^{-2}\right]'$ at $x = 1$

C. Find $\left[f(x)g^{3}(x)\right]'$ at $x = 0$

D. Find $\left[\dfrac{f(x)}{g(x)+1}\right]'$ at $x = 1$

3.6 H.W.

1. $y = \left(\dfrac{\sin x}{1+\cos(2x)}\right)^2$; Find $\dfrac{dy}{dx}$

2. $y = x^3(2x-5)^4$; Find $\dfrac{dy}{dx}$

3. $y = \sin^3 x \tan(4x)$; Find $\dfrac{dy}{dx}$

4. $y = (1+\cos^2(7x))^3$; Find $\dfrac{dy}{dx}$

5. $y = \sqrt{\tan(5x)}$; Find $\dfrac{dy}{dx}$

6. $y = (x - \sin(3x))^2$; Find $\dfrac{dy}{dx}$

7. $y = (2x + \sqrt{x})^{-2}$; Find $\dfrac{dy}{dx}$

8. Find $\dfrac{d}{dx}\left[\dfrac{1}{g^2(x)}\right]$

9. Find $\dfrac{d}{dx}\left[f(g(x)+x)\right]$

10. Find $\dfrac{d}{dx}\left[\sqrt{f^2(x)+g^2(x)}\right]$

11. Find $\dfrac{d}{dx}\left[f(x)\cdot g^3(x)\right]$

12. Find $\dfrac{d}{dx}\left[\dfrac{f^2(x)}{3x - g(h(x))}\right]$