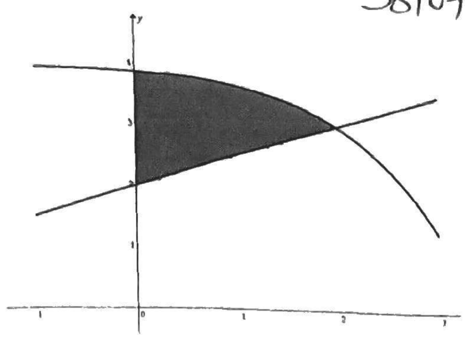
Q404. Review ASN PART 5

Solutions



1. Consider the shaded region R, in the first quadrant, bounded by $y = 2 + \frac{x}{2}$ and $y = 4 - e^{(x-2)}$.

CALCULATORS PERMITTED

A. Write, but <u>do not evaluate</u> an expression involving one or more integrals used to find the area of R.

$$A = \int_0^2 \left[4 - e^{(x-2)} - \left(2 + \frac{x}{2}\right)\right] dx$$

B. Find the volume of the solid that results in revolving region R about the line $y = 5$.

$$V = \pi \int_0^2 \left[\left(5 - \left(2 + \frac{x}{2}\right)\right)^2 - \left(5 - \left(4 - e^{(x-2)}\right)\right)^2\right] dx \approx 8.447\pi \text{ OR } 26.535$$

C. Write, but <u>do not evaluate</u> an expression involving one or more integrals used to find the volume of the solid that results in revolving region R about the line $y = 1$.

$$V = \pi \int_0^2 \left[\left(4 - e^{(x-2)} - 1\right)^2 - \left(2 + \frac{x}{2} - 1\right)^2\right] dx$$

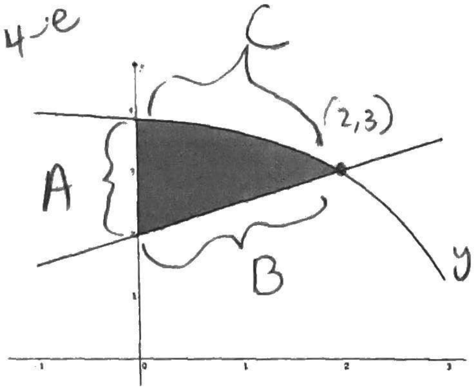
$y - 4 = -e^{x-2}$

$4 - y = e^{x-2}$

$\ln(4-y) = x - 2$

$2 + \ln(4-y) = x$

$y(0) = 4 - e^{-2}$



$y = 2 + \frac{x}{2} \rightarrow x = 2y - 4$

$y = 4 - e^{x-4} \rightarrow x = 2 + \ln(4-y)$

1. Consider the shaded region R, in the first quadrant, bounded by $y = 2 + \frac{x}{2}$ and $y = 4 - e^{(x-2)}$.

CALCULATORS PERMITTED

D. The region R is the base of a solid. For each x the cross section of the solid taken perpendicular to the x-axis is a rectangle whose base lies in R and whose height is twice its base. Write, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid.

$$A = (\text{base})(2\,\text{base}) = 2(\text{base})^2 = 2\left(4 - e^{(x-2)} - \left(2 + \frac{x}{2}\right)\right)^2$$

$$V = 2\int_0^2 \left[4 - e^{(x-2)} - \left(2 + \frac{x}{2}\right)\right]^2 dx$$

E. The region R is the base of a solid. For each y the cross section of the solid taken perpendicular to the y-axis is a square whose base lies in R. Write, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid.

$$\int_2^3 (\text{base}_1)^2\,dy + \int_3^{4-e^{(-2)}} (\text{base}_2)^2\,dy = \int_2^3 [2y-4]^2\,dy + \int_3^{4-e^{(-2)}} [2 + \ln(4-y)]^2\,dy$$

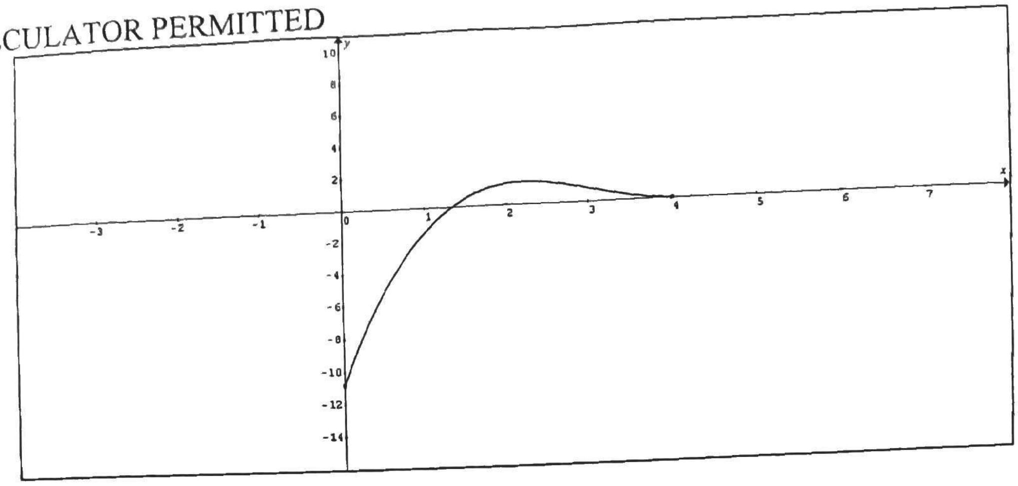
F. Write, but do not evaluate, an expression involving one or more integrals used to find the perimeter of the region R.

$$P = \left[4 - e^{(-2)} - 2\right] + \sqrt{5} + \int_0^2 \sqrt{1 + [-e^{x-2}]^2}\,dx$$

$\uparrow$

dy/dx

2. CALCULATOR PERMITTED



2. Consider the function $f(x) = 0.5x^3 - 4.675x^2 + 13.4x - 10.8$ defined on the interval $0 \le x \le 4$ as shown in the diagram above. **Find** the area bounded by $f(x)$ and the x-axis.

$$A = -\int_0^{1.35} f(x)\,dx + \int_{1.35}^{4} f(x)\,dx = 7.843$$

or

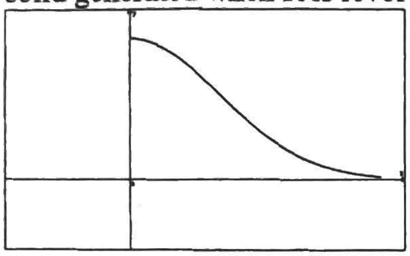
$$A = \int_0^4 |f(x)|\,dx = 7.843$$

3. (NO CALCULATOR) **Find** the length of the curve $y = \int_0^x \sqrt{t^2 - 1}\,dt$ from $x = 1$ to $x = 2$.

$$\frac{dy}{dx} = \sqrt{x^2 - 1} \quad ; \quad L = \int_1^2 \sqrt{1 + \left(\sqrt{x^2 - 1}\right)^2}\,dx$$

$$= \int_1^2 \sqrt{1 + x^2 - 1}\,dx = \int_1^2 \sqrt{x^2}\,dx = \int_1^2 x\,dx = \left.\frac{x^2}{2}\right]_1^2 = 2 - \frac{1}{2} = \boxed{\frac{3}{2}}$$

4. (NO CALCULATOR) Consider the graph of h given by $h(x) = e^{-x^2}$ for $0 \le x < \infty$.
Let R be the unbounded region in the first quadrant below the graph of h. Find the volume of the solid generated when R is revolved about the y-axis.



$$V = 2\pi \int_0^{\infty} x e^{-x^2}\,dx = 2\pi \lim_{b\to\infty} \int_0^b x e^{-x^2}\,dx = -\frac{\cancel{2}\pi}{\cancel{2}} \lim_{d\to-\infty} \int_0^d e^u\,du$$

$u = -x^2$

$du = -2x\,dx$

$dx = \dfrac{du}{-2x}$

$x = 0 \to u = 0$

$x = \infty \to u = -\infty$

$$= -\pi \lim_{d\to-\infty} \left[e^u\right]_0^d$$

$$= -\pi \lim_{d\to-\infty} \left[\cancel{e^d} - e^0\right]$$

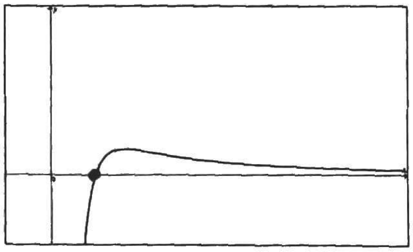
$$= -\pi\left[-1\right]$$

$$= \pi$$

(OR) $$-\pi \lim_{b\to\infty} \left[e^{-x^2}\right]_0^b = \cdots = \pi$$

5. (NO CALCULATOR)

Find the area of the region in the first quadrant that lies under the curve $y = \dfrac{\ln x}{x^2}$.



$y(1) = 0$

$$\int_1^{\infty} \frac{\ln x}{x^2}\,dx = \lim_{b\to\infty} \int_1^{b} \frac{\ln x}{x^2}\,dx = \lim_{b\to\infty} \left[\frac{-\ln x}{x} - \frac{1}{x} \right]_1^{b} \ *$$

$*$

$u = \ln x \quad dv = x^{-2}$

$du = \frac{1}{x} \quad v = -x^{-1}$

$$-\frac{\ln x}{x} + \int \frac{1}{x^2}\,dx$$

$$-\frac{\ln x}{x} - \frac{1}{x}$$

$$= \lim_{b\to\infty} \left[\frac{-\ln b}{b} \right] - \cancel{\lim_{b\to\infty} \frac{1}{b}}^{\,0} + 1$$

$\downarrow$ L'HPT

$$= \lim_{b\to\infty} \frac{-\frac{1}{b}}{1} + 1$$

$$= 0 + 1$$

$$= \boxed{1}$$

:)

6. (NC) Use the integral test to determine whether $\sum_{2}^{\infty} \frac{1}{n \ln n}$ converges or diverges. Show work.

$$a_n = \frac{1}{n \ln n} \qquad f(x) = \frac{1}{x \ln x}$$

$$\int_2^{\infty} f(x)\,dx = \int_2^{\infty} \frac{1}{x \ln x}\,dx = \lim_{b \to \infty} \int_2^{b} \frac{1}{x \ln x}\,dx = \lim_{m \to \infty} \int_{\ln 2}^{m} \frac{du}{u}$$

$u = \ln x$

$du = \frac{dx}{x}$

$dx = x\,du$

$x = 2 \to u = \ln 2$

$x = \infty \to u = \infty$

$$= \lim_{m \to \infty} \ln|u| \Big]_{\ln 2}^{m}$$

$$= \lim_{m \to \infty} \left[\ln|m| - \ln|\ln 2| \right]$$

∞

$$= \infty$$

$\therefore \sum a_n = \sum_{2}^{\infty} \frac{1}{n \ln n}$ diverges also by the I.T.