



BC.Q404.REVIEW ASSESSMENT (PART 3)

THE FUNDMENTAL THEOREM OF CALCULUS
(20 points)

NO CALCULATOR ANSWERS SHEET

NAME:

KEY

DATE:

BLOCK:

I (*print name*) _____ certify that I wrote **all** marks made in this assessment. I did not write **anything** that I do not fully understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

QUESTION 1

A. Find $g(2)$, $g'(2)$, and $g''(2)$.

$$g(2) = \int_{-1}^2 f(x) dx = \left[\frac{1+3}{2}(2) + \frac{1}{2}(3) \right] = 4 + \frac{3}{2} = \frac{11}{2}$$

$$g'(2) = f(2) = 0 \leftarrow \text{point on graph of } f$$

$$g''(2) = f'(2) = -3 \leftarrow \text{slope of graph of } f$$

B. On what interval between $-5 < x < 5$ is g increasing? Justify your answer.

$$(-5, 2] \text{ or } (-5, 2) \text{ because } g'(x) = f(x) > 0 \text{ on } (-5, 2)$$

C. For what values of x is $g(x)$ concave downward? Justify your answer.

$$(-3, -1) \cup (1, 3) \text{ because } g''(x) = f'(x) < 0 \text{ on this interval}$$

D. Write the equation of any horizontal tangents to $g(x)$ between $-5 < x < 5$.

$$g'(x) = f(x) = 0 \text{ at } x = 2 \quad g(2) = \frac{11}{2} \quad y - \frac{11}{2} = 0 \text{ (} x = 2 \text{)}$$

$y = \frac{11}{2}$

E. Find the absolute minimum value of $g(x)$ on the interval $-5 \leq x \leq 5$. Justify your answer.

$$g'(x) = 0 \text{ at } x = 2 \leftarrow \text{local max here}$$

you must communicate this

$$g(5) = \int_{-1}^5 f(x) dx = 4 + \frac{3}{2} - \frac{3}{2} - 6 = -2$$

$$g(-5) = \int_{-1}^{-5} f(x) dx = - \int_{-5}^{-1} f(x) dx = - \left[4 + \frac{2+1}{2} \cdot 2 \right] = -7 \leftarrow \text{absolute min}$$

F. Find the average rate of change of $g'(x)$ on $-5 \leq x \leq 5$. Does the Mean Value Theorem applied on the interval $-5 \leq x \leq 5$ guarantee a value of c , for $-5 < c < 5$, such that $g''(c)$ is equal to this average rate of change $g'(x)$? Why or why not? If so, find a value of c that satisfies the conclusion of the mean value theorem.

$$\begin{aligned} \text{ave rate } \Delta \text{ of } g'(x) &= \text{ave. rate } \Delta \text{ of } f(x) = \frac{f(5) - f(-5)}{5 - (-5)} \\ &= \frac{-3 - 2}{10} = \frac{-5}{10} = -\frac{1}{2} \end{aligned}$$

The mean value theorem does not guarantee b/c $f(x)$ is not differentiable on $(-5, 5)$. Namely, $f(x)$ has a corner at $x = -3, -1, 1, 3$.

In order to guarantee $f(x)$ must be both continuous on $[-5, 5]$ and differentiable on $(-5, 5)$.

G. Find the average value of $f(x)$ on $-5 \leq x \leq 5$. Does the Mean Value Theorem applied on the interval $-5 \leq x \leq 5$ guarantee a value of z , for $-5 < z < 5$, such that $f'(z)$ is equal to this average value of $f(x)$? Why or why not? If so, find a value of z that satisfies the conclusion of the mean value theorem.

$$\begin{aligned} \text{Ave. value of } f(x) &= \frac{\int_{-5}^5 f(x) dx}{5 - (-5)} = \frac{1}{10} \int_{-5}^5 f(x) dx \\ &= \frac{1}{10} \left[4 + 3 + 4 + \frac{3}{2} - \frac{3}{2} - 6 \right] = \frac{5}{10} = \frac{1}{2} \end{aligned}$$

The mean value theorem (for integrals) does guarantee b/c $f(x)$ is continuous on $[-5, 5]$ (This is the only requirement for M.V.T for Integrals)

Now: $\underbrace{-3x + 6}_{\text{equation of line segment}} = \frac{1}{2} \quad x = \frac{11}{6} \quad \boxed{z = \frac{11}{6}}$

H. Let $h(x) = 2x^2 - \int_{-1}^x f(t) dt$. Find $h'(3)$.

$$h'(x) = 4x - f(x)$$

$$h'(3) = 4(3) - f(3) = 12 - (-3) = \boxed{15}$$

I. Let $p(x) = \int_{-1}^{4x^3+2} f(t) dt$. Find $p'(-1)$.

$$p'(x) = f(4x^3 + 2) (12x^2)$$

$$p'(-1) = f(-2) \cdot 12(-1)^2 = \frac{3}{2} (12) = \boxed{18}$$

QUESTION 2

A. Estimate $g(8)$ using a right Riemann sum with three rectangles.

$$g(8) = \int_0^8 f(x) dx \approx 14(2) + 12(5) + 10(1) = 98$$

B. Estimate $g(-5)$ using a trapezoidal approximation with three trapezoids.

$$\begin{aligned} g(-5) &= \int_0^{-5} f(x) dx = - \int_{-5}^0 f(x) dx = - \left[\frac{-10 + (-3)}{2} (3) + \frac{(-3) + 4}{2} (1) + \frac{4 + 8}{2} (1) \right] \\ &= - \left[\frac{-13}{2} \cdot 3 + \frac{1}{2} + 6 \right] \end{aligned}$$

C. Find $g'(-1)$

$$g'(-1) = f(-1) = 4$$

D. Estimate $g''(2.6)$.

$$g''(2.6) = f'(2.6) \approx \frac{f(7) - f(2)}{7 - 2} = \frac{12 - 14}{5} = \frac{-2}{5}$$

E. If $m(x) = \int_0^{1-\ln x} f(t) dt$, find $m'(e)$.

$$m'(x) = f(1 - \ln x) \cdot \frac{-1}{x} \quad m'(e) = f(1 - \ln e) \cdot \frac{-1}{e} = f(0) \cdot \frac{-1}{e} = \frac{-8}{e}$$

F. If $b(x) = g\left(\frac{x}{4}\right)$, find $b'(8)$.

$$b(x) = g\left(\frac{x}{4}\right) = \int_0^{\frac{x}{4}} f(t) dt \quad b'(x) = f\left(\frac{x}{4}\right) \cdot \frac{1}{4} \quad b'(8) = f(2) \cdot \frac{1}{4} = \frac{14}{4} = \frac{7}{2}$$

G. What is the minimum number of times that $f'(x) = 0$ on $-5 \leq x \leq 8$. Make use of the appropriate theorems to justify your answer.

Minimum of one time. $f(x)$ is continuous on $[-5, 8]$ ✓
 $f(x)$ is differentiable on $(-5, 8)$ ✓

on $[0, 2]$ ave $f'(x) = \frac{6}{2} = 3$ $f'(x) = 3$ at some point on $(0, 2)$ by M.V.T

on $[2, 7]$ ave $f'(x) = \frac{-2}{5}$ $f'(x) = \frac{-2}{5}$ at some point on $(2, 7)$ by M.V.T
 $f'(x)$ is continuous on $[0, 7]$

$f'(x)$ goes from positive to negative at least on time on $[0, 7]$

$\therefore$ by I.V.T $f'(x) = 0$ at least on time.

QUESTION 3

A. The graph of $y = f(x)$ is decreasing for what x -values?

$$[-3, -1] \cup (1, 5] \quad \text{or} \quad (-3, -1) \cup (1, 5)$$

B. Compute $f(5)$ and $f(-3)$

$$\int_0^5 f'(x) dx = f(5) - f(0) \rightarrow (1 - 2 - 1) = f(5) - 12 \quad \therefore f(5) = \boxed{12 + 1 - 2 - 1} = 10$$

$$\int_{-3}^0 f'(x) dx = f(0) - f(-3) \rightarrow f(-3) = \boxed{12 - \left(-\frac{1}{3} + 2 - 3\right) - \frac{1}{2}} = \frac{77}{6}$$

$$f(-3) = f(0) - \int_{-3}^0 f'(x) dx$$

$$\int_{-3}^{-1} f'(x) dx = \int_{-3}^{-1} (x^2 + 4x + 3) dx = \left[\frac{x^3}{3} + 2x^2 + 3x \right]_{-3}^{-1} = \left(-\frac{1}{3} + 2 - 3\right) - (-9 + 18 - 9)$$

$$\int_{-1}^0 f'(x) dx = \frac{1}{2} \leftarrow \text{area of triangle}$$

C. Find the absolute maximum and absolute minimum values of $f(x)$ on $-3 \leq x \leq 5$. Justify your answers.

Interior critical x values :

$$f'(x) = 0 \text{ at } x = -1$$

$$f'(x) \text{ DNE at } x = 1$$

you must state this ...
it justifies why they are critical x-values.

endpoint x values $x = -3, x = 5$

$$f(-3) = \frac{71}{6}$$

$$f(-1) = f(0) - \int_{-1}^0 f'(x) dx = 12 - \frac{1}{2} = \frac{23}{2}$$

$$f(1) = f(0) + \int_0^1 f'(x) dx = 12 + 1 = 13$$

$$f(5) = 10$$

abs max = 13
abs min = 10

D. Find $\lim_{x \rightarrow 0} \frac{f(x) - 12}{1 - \ln(x + e)}$ Show work.

$$\lim_{x \rightarrow 0} \frac{f(x) - 12}{1 - \ln(x + e)} \quad \left\{ \frac{0}{0} \right\} = \lim_{x \rightarrow 0} \frac{f'(x)}{-\frac{1}{x+e}} = \frac{f'(0)}{-\frac{1}{e}} = \frac{1}{-\frac{1}{e}} = \boxed{-e}$$

L'HOPITAL'S RULE

$$\lim_{x \rightarrow 0} f(x) - 12 = 0$$

also

$$\lim_{x \rightarrow 0} 1 - \ln(x + e) = 0$$

E. Find $\lim_{x \rightarrow 1^+} \frac{f'(x) + x}{x^2 - 1}$ Show work.

$$\lim_{x \rightarrow 1^+} \frac{f'(x) + x}{x^2 - 1} \quad \left\{ \frac{0}{0} \right\} = \lim_{x \rightarrow 1^+} \frac{f''(x) + 1}{2x} = \frac{0 + 1}{2} = \boxed{\frac{1}{2}}$$

L'HOPITAL'S RULE

$$\lim_{x \rightarrow 1^+} f'(x) + x = 0$$

also

$$\lim_{x \rightarrow 1^+} x^2 - 1 = 0$$