Advanced Calculus

SOLUTIONS

BC.Q404.REVIEW ASSESSMENTS (PART 1)

THE EARLY CHAPTERS

(X points)

NO CALCULATORS

NAME:

DATE:

BLOCK:

I *(print name)*__________________________________ certify that I wrote and fully understand **all** marks made in this assessment. I did not write anything that I do not understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

1. Let $f(x)=\begin{cases}5-(1-x)^{5/2}\ ; x<1\\ 6-(2-x)^2\ ; x\ge 1\end{cases}$

(a) Is f continuous at $x=1$? Why or why not?

(b) Find the absolute maximum and the absolute minimum value of f on the closed interval $-1\le x\le 4$. Box your answer. Show the analysis that leads to your conclusion.

a) i) $f(1) = 6-(2-1)^2 = 5$

ii) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 6-(2-x)^2 = 5$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 5-(1-x)^{5/2} = 5$

$\therefore \lim_{x\to 1} f(x) = 5$ * It is not sufficient to just say the limit exists

iii) $\lim_{x\to 1} f(x) = f(1)$ $\therefore$ f is continuous at $x=1$

b) $f'(x) = \begin{cases} 5/2\,(1-x)^{3/2} & \text{for } x<1 \\ 2(2-x) & \text{for } x>1\end{cases}$ * $f'(x)$ DNE at $x=1$

but the function f(x) is continuous at $x=1$

interior critical x-values: $f'(x)=0$ at $x=2$ $f'(x)$ DNE at $x=1$

endpoint: $x=-1$, $x=4$

$f(-1) = 5-2^{5/2}$

$f(1) = 5$

$f(2) = 6$

$f(4) = 2$

$\boxed{\begin{array}{l}\text{absolute max} = 6 \\ \text{absolute min} = 5-2^{5/2}\end{array}}$ or $5-\sqrt{32}$ or $5-4\sqrt{2}$

multiplication ... not addition

2. Use Lagrange (*prime*) notation to express the following derivatives

A. $[f(g(x)+2)]' = f'(g(x)+2)\cdot g'(x)$

B. $[\ln(f(x))+g(2x)]' = \frac{1}{f(x)}\cdot f'(x) + g'(2x)\cdot 2$

$[f^2(x)] = [f(x)]^2$

C. $[f^2(x)\cdot h(g(x))]' = f^2(x)\cdot h'(g(x))\cdot g'(x) + h(g(x))\cdot 2f(x)\cdot f'(x)$

Product Rule

3. Let $f(x) = -x^5 - x^3 - x - 5$ and $g(x)$ be the inverse function of *f*. Find $g'(-2)$.

$g'(2) = \frac{1}{f'(-1)} = \frac{1}{-5(-1)^4 - 3(-1)^2 - 1} = \frac{-1}{9}$

$-2 = -a^5 - a^3 - a - 5$

$3 = -a^5 - a^3 - a$

guess and check

$a = -1$

4. Find the derivative of $\sin^{-1}(2x^2)$

Book of memories

$\frac{1}{\sqrt{1-(2x^2)^2}}\cdot 4x$

Product Rule

$(x \cdot y)$

5. Consider the curve C: $\sin(xy) + 2y = x + y^2 + \frac{4-\pi}{4}$

A. Find $\frac{dy}{dx}$.

B. Find the equation of the line tangent to the curve at the point $\left(\frac{\pi}{4}, 2\right)$

$$\cos(xy) \cdot \left[x \frac{dy}{dx} + y(1) \right] + 2 \frac{dy}{dx} = 1 + 2y \frac{dy}{dx}$$

$$x \cos(xy) \frac{dy}{dx} + y \cos(xy) + 2 \frac{dy}{dx} = 1 + 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} \left(x \cos(xy) + 2 - 2y \right) = 1 - y \cos(xy)$$

$$\frac{dy}{dx} = \frac{1 - y \cos(xy)}{x \cos(xy) + 2 - 2y}$$

$$\left. \frac{dy}{dx} \right|_{\left(\frac{\pi}{4}, 2\right)} = \frac{1}{2-4} = -\frac{1}{2}$$

$$\boxed{y - 2 = -\frac{1}{2}\left(x - \frac{\pi}{4}\right)}$$

6. Consider the curve C: $\sqrt{3}\,y + 2\sin y = 1 + x^3$ for $0 \le y \le \pi$.
Find a point on the graph of C where the tangent line to C is vertical.

$$\sqrt{3}\,\frac{dy}{dx} + 2\cos(y)\,\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{\sqrt{3} + 2\cos y}$$ vertical means $\frac{dy}{dx}$ DNE

set denominator = zero

$\sqrt{3} + 2\cos y = 0$ $\quad \cos y = \frac{-\sqrt{3}}{2}$ $\quad y = \frac{5\pi}{6}$

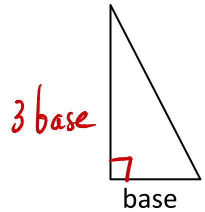
Now find x ...

$$\sqrt{3}\,\frac{5\pi}{6} + 2\sin\left(\frac{5\pi}{6}\right) = 1 + x^3$$

$x^3 = \frac{5\sqrt{3}\pi}{6}$ $\quad x = \sqrt[3]{\frac{5\sqrt{3}\pi}{6}}$

$$\left(\sqrt[3]{\frac{5\sqrt{3}\,\pi}{6}},\ \frac{5\pi}{6}\right)$$

7. A chemical leak in the corner of a science laboratory room makes the shape of a right triangle on the floor. The triangle grows in such a way that the height is always three times the base. The length of the base is growing at the rate 7/10 feet per second at the very instant the base is 5/3 feet in length.



Given | $\frac{db}{dt} = \frac{7}{10}$ ft/sec when $b = \frac{5}{3}$

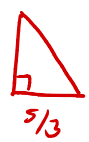
A. Find the rate at which the area of the triangle is growing at the instant the base is 5/3 feet in length. Include the units of measure.

Find | $\frac{dA}{dt}$

Rel | $A = \frac{1}{2}(b)(3b) = \frac{3b^2}{2}$

$\frac{dA}{dt} = 3b\frac{db}{dt}$

$\left.\frac{dA}{dt}\right|_{\boxed{\cdot}} = 3\left(\frac{5}{3}\right)\left(\frac{7}{10}\right) = \frac{7}{2}$ ft²/sec



B. Find the rate at which the hypotenuse of the right triangle is growing at the instant the base is 5/3 feet in length. Include the units of measure.

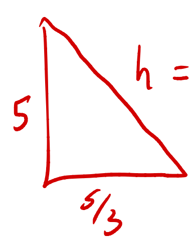
Find | $\frac{dh}{dt}$

Rel | $b^2 + (3b)^2 = h^2$

$b^2 + 9b^2 = h^2$

$10b^2 = h^2$

$20b\frac{db}{dt} = 2h\frac{dh}{dt}$



$h = \sqrt{25 + \frac{25}{9}}$

$2\left(\frac{5}{3}\right)\left(\frac{7}{10}\right) = 2\left(\sqrt{25 + \frac{25}{9}}\right)\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{20\left(\frac{5}{3}\right)\left(\frac{7}{10}\right)}{2\sqrt{25 + \frac{25}{9}}} = \frac{\frac{35}{3}}{\frac{5\sqrt{10}}{3}} = \frac{7}{\sqrt{10}}$ ft/sec

or $\frac{7\sqrt{10}}{10}$

C. The rate of change of the total temperature of the triangle is modeled by the function $T'(t) = t\left(t^2+1\right)^4$ units per second. Based on this model, what was the temperature increase from time one to two seconds?

$$\int_1^2 T'(t)\,dt = \int_1^2 t(t^2+1)^4\,dt = \frac{1}{2}\int_2^5 u^4\,du$$

$u = t^2+1$

$du = 2t\,dt$

$dt = \dfrac{du}{2t}$

$$= \frac{1}{2}\left[\frac{u^5}{5}\right]_2^5$$

$$= \frac{1}{10}(t^2+1)^5\Big]_1^2$$

$$= \boxed{\frac{1}{10}\left[(5)^5-(2)^5\right]}$$

$$= \frac{3093}{10} \text{ units}$$