BC.Q404.REVIEW ASSESSMENTS (Part 6)

CH 9 (REVISITED) – Sequence and Series

(20 points)

NO CALCULATOR

NAME:

DATE:

BLOCK:

I (*print name*)_____ certify that I wrote and fully understand **all** marks made in this assessment. I did not write anything that I do not understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

1. Let f be the function given by $f(x) = \sin\left(5x + \frac{\pi}{4}\right)$, and let $P(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

(a) Find $P(x)$

(b) Find the coefficient of x^{22} in the Taylor series for f about $x = 0$.

(c) Use the Lagrange error bound to show $\left| f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right) \right| < \frac{1}{100}$

(d) Let G be the function given by $G(x) = \int_0^x f(t)dt$. Write the third degree Taylor polynomial for G about $x = 0$.

2.

The Maclaurin series for e^x is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$. The continuous function f is defined by $f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \neq 1$ and $f(1) = 1$. The function f has derivatives of all orders at $x = 1$.

(a) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x = 1$.

(b) Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

(c) Use the ratio test to find the interval of convergence for the Taylor series found in part (b).

(d) Use the Taylor series for f about $x = 1$ to determine whether the graph of f has any points of inflection.

3. Let f be the function given by $f(x) = e^{-x^2}$.

A. Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

B. Use your answer to part (a) to find $\lim_{x \to 0} \dfrac{1 - x^2 - f(x)}{x^4}$

C. Write the first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2}\,dt$ about $x = 0$. Use the first two terms of your answer to estimate $\int_0^{1/2} e^{-t^2}\,dt$.

D. Explain why the estimate found in part (C) differs from the actual value of $\int_0^{1/2} e^{-t^2}\,dt$ by less than $\dfrac{1}{200}$.

4.

Let f be a function with derivatives of all orders and for which $f(2) = 7$. When n is odd, the nth derivative of f at $x = 2$ is 0. When n is even and $n \geq 2$, the nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \dfrac{(n-1)!}{3^n}$.

(a) Write the sixth-degree Taylor polynomial for f about $x = 2$.

(b) In the Taylor series for f about $x = 2$, what is the coefficient of $(x-2)^{2n}$ for $n \geq 1$?

(c) Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.