BC.Q404.REVIEW ASSESSMENT (PART 3)

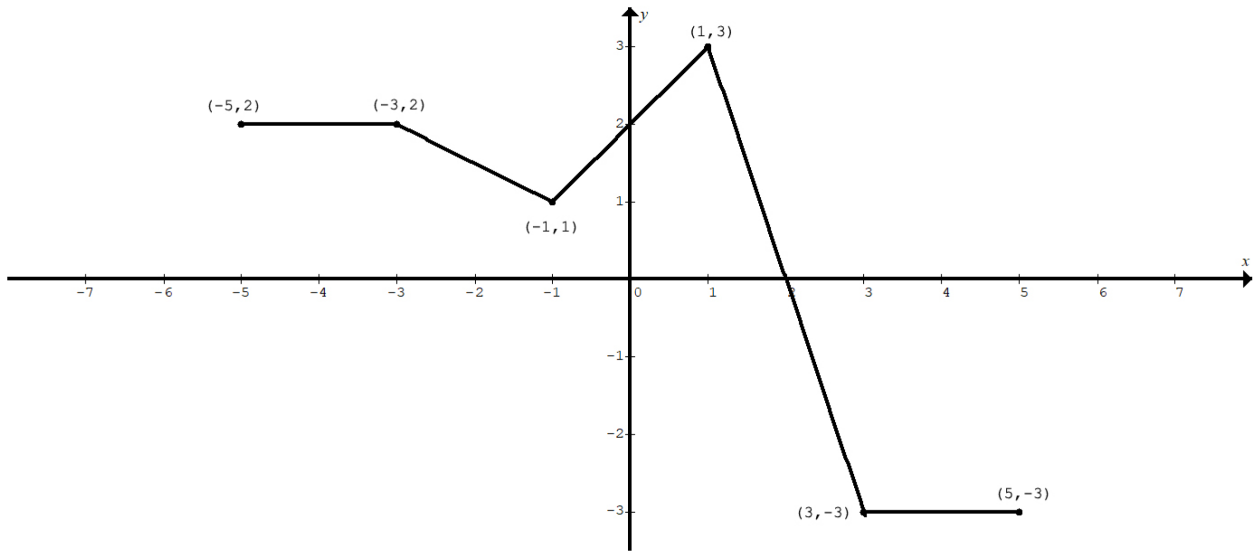
THE FUNDAMENTAL THEOREM OF CALCULUS

(20 points)

NO CALCULATOR

QUESTION SHEET

PLEASE PUT ALL WORK AND ANSWERS ON THE ANSWER SHEET. PLEASE DO NOT HAND IN THE QUESTION SHEET.



1. The graph of the function f above consists of three line segments.

Let g be the function given by $g(x) = \int_{-1}^{x} f(t)dt$

A. Find $g(2)$, $g'(2)$, and $g''(2)$.

B. On what interval between $-5 < x < 5$ is g increasing? Justify your answer.

C. For what values of x is $g(x)$ concave downward? Justify your answer.

D. Write the equation of any horizontal tangents to $g(x)$ between $-5 < x < 5$.

E. Find the absolute minimum value of $g(x)$ on the interval $-5 \le x \le 5$. Justify your answer.

F. Find the average rate of change of $g'(x)$ on $-5 \le x \le 5$. Does the Mean Value Theorem applied on the interval $-5 \le x \le 5$ guarantee a value of c, for $-5 < c < 5$, such that $g''(c)$ is equal to this average rate of change $g'(x)$? Why or why not? If so, find a value of c that satisfies the conclusion of the mean value theorem.

G. Find the average value of $f(x)$ on $-5 \le x \le 5$. Does the Mean Value Theorem applied on the interval $-5 \le x \le 5$ guarantee a value of z, for $-5 < z < 5$, such that $f(z)$ is equal to this average value of $f(x)$? Why or why not? If so, find a value of z that satisfies the conclusion of the mean value theorem.

H. Let $h(x) = 2x^2 - \int_{-1}^{x} f(t)dt$. Find $h'(3)$.

I. Let $p(x) = \int_{-1}^{4x^3+2} f(t)dt$. Find $p'(-1)$.

x	-5	-2	-1	0	2	7	8
$f(x)$	-10	-3	4	8	14	12	10

2. $y = f(x)$ is a continuous and differentiable function. Select values for $f(x)$ are given in the table above.

Let $g(x) = \int_0^x f(t)dt$ for $-5 \le x \le 8$.

A. Estimate $g(8)$ using a right Riemann sum with three rectangles.

B. Estimate $g(-5)$ using a trapezoidal approximation with three trapezoids.

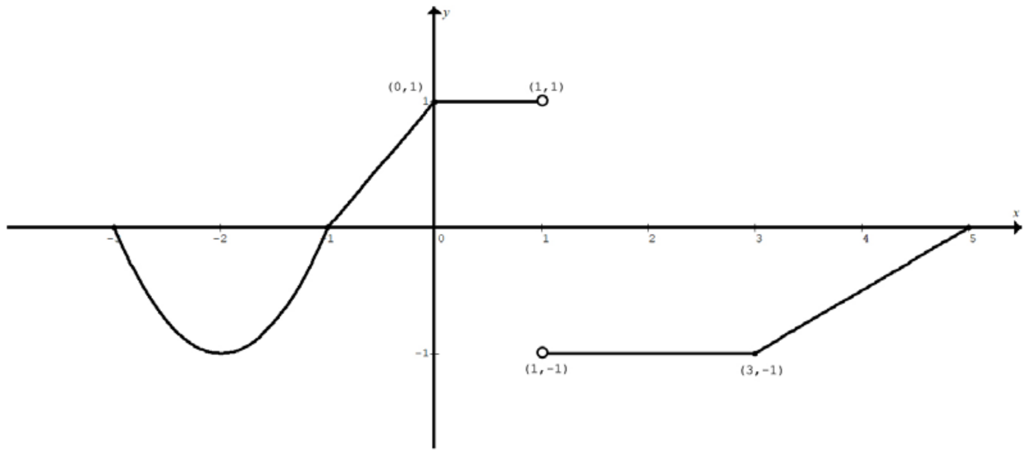
C. Find $g'(-1)$

D. Estimate $g''(2.6)$.

E. If $m(x) = \int_0^{1-\ln x} f(t)dt$, find $m'(e)$.

F. If $b(x) = g\left(\frac{x}{4}\right)$, find $b'(8)$.

G. What is the minimum number of times that $f'(x) = 0$ on $-5 \le x \le 8$. Make use of the appropriate theorems to justify your answer.



3. Above is the graph of $y = f'(x)$. This derivative graph consists of the parabola $y = x^2 + 4x + 3$ on $-3 \le x \le -1$ and a series of line segments on $-1 \le x \le 5$.
The graph of $y = f(x)$ (whose graph is NOT shown) is continuous on $-3 \le x \le 5$.
Suppose that $f(0) = 12$.

A. The graph of $y = f(x)$ is decreasing for what *x*-values?
B. Compute $f(5)$ and $f(-3)$
C. Find the absolute maximum and absolute minimum values of $f(x)$ on $-3 \le x \le 5$. Justify your answers.
D. Find $\lim_{x \to 0} \dfrac{f(x) - 12}{1 - \ln(x + e)}$ Show work.
E. Find $\lim_{x \to 1^+} \dfrac{f'(x) + x}{x^2 - 1}$ Show work.

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ANSWERS SHEET

NAME:

DATE:

BLOCK:

I (*print name*)________________________________ certify that I wrote **all** marks made in this assessment. I did not write **anything** that I do not fully understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

QUESTION 1

A. Find $g(2)$, $g'(2)$, and $g''(2)$.

B. On what interval between $-5 < x < 5$ is g increasing? Justify your answer.

C. For what values of x is $g(x)$ concave downward? Justify your answer.

D. Write the equation of any horizontal tangents to $g(x)$ between $-5 < x < 5$.

E. Find the absolute minimum value of $g(x)$ on the interval $-5 \le x \le 5$. Justify your answer.

F. Find the average rate of change of $g'(x)$ on $-5 \le x \le 5$. Does the Mean Value Theorem applied on the interval $-5 \le x \le 5$ guarantee a value of c, for $-5 < c < 5$, such that $g''(c)$ is equal to this average rate of change $g'(x)$? Why or why not? If so, find a value of c that satisfies the conclusion of the mean value theorem.

G. Find the average value of $f(x)$ on $-5 \le x \le 5$. Does the Mean Value Theorem applied on the interval $-5 \le x \le 5$ guarantee a value of z, for $-5 < z < 5$, such that $f(z)$ is equal to this average value of $f(x)$? Why or why not? If so, find a value of z that satisfies the conclusion of the mean value theorem.

H. Let $h(x) = 2x^2 - \int_{-1}^{x} f(t)dt$. Find $h'(3)$.

I. Let $p(x) = \int_{-1}^{4x^3+2} f(t)dt$. Find $p'(-1)$.

QUESTION 2

A. Estimate $g(8)$ using a right Riemann sum with three rectangles.

B. Estimate $g(-5)$ using a trapezoidal approximation with three trapezoids.

C. Find $g'(-1)$

D. Estimate $g''(2.6)$.

E. If $m(x) = \int_{0}^{1-\ln x} f(t)dt$, find $m'(e)$.

F. If $b(x) = g\left(\frac{x}{4}\right)$, find $b'(8)$.

G. What is the minimum number of times that $f'(x) = 0$ on $-5 \le x \le 8$. Make use of the appropriate theorems to justify your answer.

QUESTION 3

A. The graph of $y = f(x)$ is decreasing for what x-values?

B. Compute $f(5)$ and $f(-3)$

C. Find the absolute maximum and absolute minimum values of $f(x)$ on $-3 \le x \le 5$. Justify your answers.

D. Find $\lim_{x\to 0} \frac{f(x)-12}{1-\ln(x+e)}$ Show work.

E. Find $\lim_{x\to 1^+} \frac{f'(x)+x}{x^2-1}$ Show work.