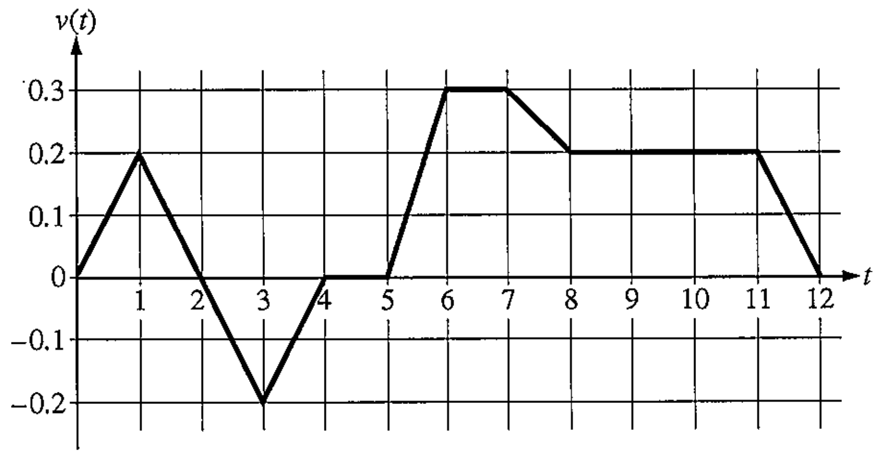
 # 1

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

(a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

(b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\, dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\, dt$.

(c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

(d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

#2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

 #3

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

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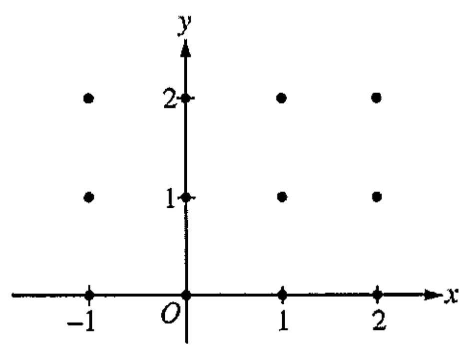
4. Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

   (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(0, 1)$.

   (Note: Use the axes provided in the pink test booklet.)



   (b) The solution curve that passes through the point $(0, 1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

   (c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

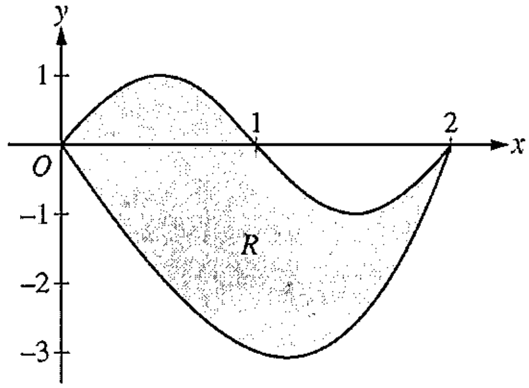
   (d) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

WRITE ALL WORK IN THE TEST BOOKLET.

#4

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

   (a) Find the area of R.

   (b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

   (d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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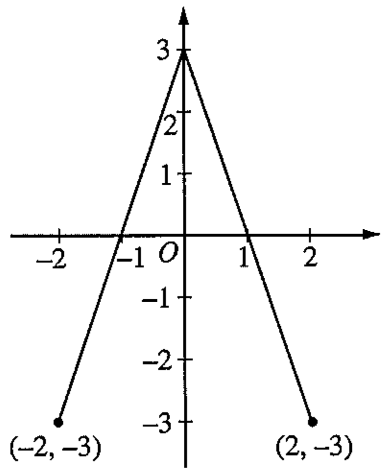
 # 5

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

   (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

   (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

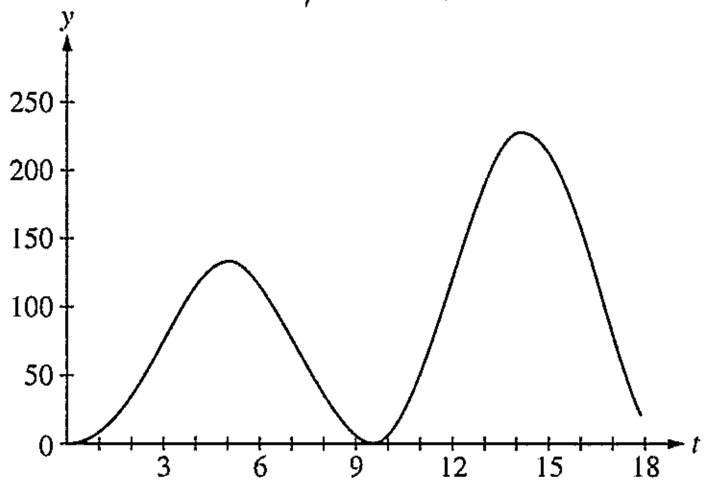
   (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

   (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
   (Note: The axes are provided in the pink test booklet only.)

   make your own axes

#6

yes – calculator



2. At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

   (a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

   (b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

   (c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

PREPACK #7

2005 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

   (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

   (c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

   (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

#8

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the x-axis.

   (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

 #9

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

4. Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

   (a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.

   (b) At the point $(-1, 2)$, the value of $\frac{d^2y}{dx^2}$ is -12. Find the second-degree Taylor polynomial for f about $x = -1$.

   (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

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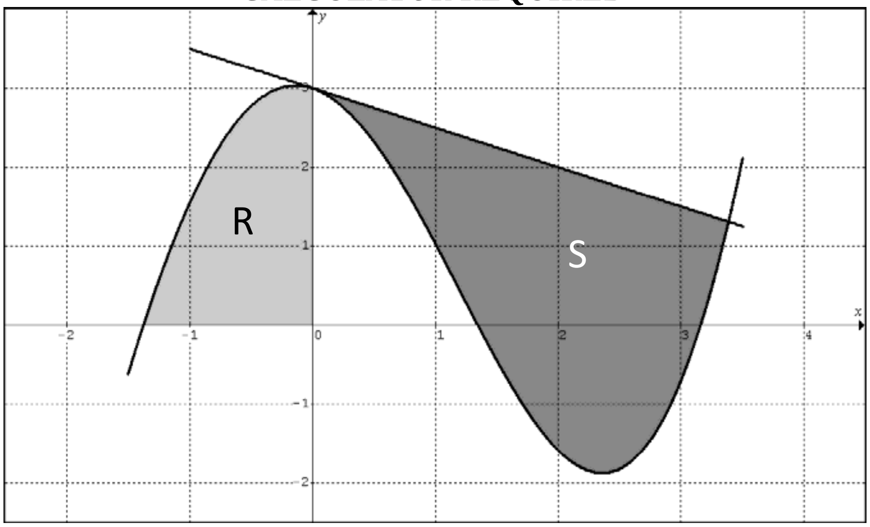
REVIEW PREPACKAGE
PROBLEM #10
AP YEAR: MIX/MADE AP#MIX

-----CALCULATOR REQUIRED-----



1. Let f be the function $f(x)=\frac{x^3}{4}-\frac{x^2}{3}-\frac{x}{2}+3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and the line ℓ, the line tangent to the graph of f at $x=0$, as shown above.

(a) Find the area of region R.
(b) Find the volume of the solid generated when R is rotated about the horizontal line $y=-2$.
(c) The region R is the base of a solid. Every cross section perpendicular to the *x*-axis is an equilateral triangle whose side lies flat on R. Write, but do not evaluate, an expression involving one or more integrals that can be used to find the volume of the solid.
(d) Write, but do not evaluate an expression involving one or more integrals used to find the perimeter of region R.
(e) Write, but do not evaluate, an integral expression that can be used to find the area of S.
(f) Consider the function $g(x)=\int_{-1}^{x} f(t)\,dt$. Find the length of g on $-1\le x\le 0$.