BC: Q303 CH8. Lesson 1:

Chapter 8.2 Indeterminate Forms and L'Hopital's Rule Reference Sheet

Indeterminate Form	Limit Form $\lim_{x \to c} \dfrac{f(x)}{g(x)}$
$\dfrac{0}{0}$	$\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = 0$
$\dfrac{\infty}{\infty}$	$\lim_{x \to c} f(x) = \infty$ or $-\infty$ and $\lim_{x \to c} g(x) = \infty$ or $-\infty$

L'Hopital's Rule: Suppose that f and g are differentiable on an open interval (a, b) containing c, except possibly at c itself. If $f(x)/g(x)$ has the indeterminate form $0/0$ or ∞/∞ at $x = c$ and if $g'(x) \neq 0$ for $x \neq c$, then $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \lim_{x \to c} \dfrac{f'(x)}{g'(x)}$ provided the limit exist or equals 0.

Indeterminate Form	Limit Form $\lim_{x \to c}[f(x)g(x)]$
$0 \cdot \infty$	$\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = \infty$ or $-\infty$

Guidelines:

1. Write $f(x)g(x)$ as $\dfrac{f(x)}{1/g(x)}$ or $\dfrac{g(x)}{1/f(x)}$
2. Apply L'Hopital's Rule for $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$

Indeterminate Form	Limit Form $\lim_{x \to c} f(x)^{g(x)}$
0^0	$\lim_{x \to c} f(x) = 0$ and $\lim_{x \to c} g(x) = 0$
∞^0	$\lim_{x \to c} f(x) = \infty$ or $-\infty$ and $\lim_{x \to c} g(x) = 0$
1^∞	$\lim_{x \to c} f(x) = 1$ and $\lim_{x \to c} g(x) = \infty$ or $-\infty$

Guidelines:

1. Let $y = f(x)^{g(x)}$
2. Take the natural log of both sides
   $\ln y = \ln f(x)^{g(x)} = g(x) \ln f(x)$
3. Investigate $\lim_{x \to c} \ln y = \lim_{x \to c}[g(x) \ln f(x)]$

Conclude:

a) If $\lim_{x \to c} \ln y = L$, then $\lim_{x \to c} y = e^L$
b) If $\lim_{x \to c} \ln y = \infty$, then $\lim_{x \to c} y = \infty$
c) If $\lim_{x \to c} \ln y = -\infty$, then $\lim_{x \to c} y = 0$

BC Q303 LESSON 1 LEVEL 1 and 2 HW – L'HOPITAL'S RULE

L'HÔPITAL'S RULE PRACTICE SET # 1

1) Find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

2) Find $\lim_{x \to \pi} \frac{\csc x}{1 + \cot x}$

3) Find $\lim_{x \to 0^+} x \ln x$

4) Find $\lim_{x \to 0} \frac{\sin(x^2)}{x}$

5) Find $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x$

6) Find $\lim_{x \to 1} \frac{x^3 - 1}{4x^3 - x - 3}$

7) Find $\lim_{x \to \infty} \frac{\log_2 x}{\log_2 (x+3)}$

* 8) Find $\lim_{x \to \infty} \frac{3x - 5}{2x^2 - x + 2}$

* 9) Find $\lim_{x \to \infty} \frac{4x^3 - 2x^2 + 5}{3x^3 - x}$

* 10) Find $\lim_{x \to \infty} \frac{3x^2 + 1}{5x}$

* CAN DO WITHOUT L'HOPITAL'S RULE

BC Q303 LESSON 1 – L'HOPITAL'S RULE (Level 1 and 2)ADDITIONAL PRACTICE

L'HOPITAL'S RULE PRACTICE SET #2

① $\lim_{x \to 0} \frac{\sin x}{2x}$

② $\lim_{x \to 0} \frac{5x}{\tan x}$

③ $\lim_{x \to 5} \frac{\sqrt{x-1} - 2}{x^2 - 25}$

④ $\lim_{x \to -3} \frac{x^2 + 2x - 3}{2x^2 + 3x - 9}$

⑤ $\lim_{x \to 0} \frac{\sin x - x}{\tan x - x}$

⑥ $\lim_{x \to 0} \frac{x + 1 - e^x}{x^2}$

⑦ $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

⑧ $\lim_{x \to \pi/2} \frac{1 - \sin x}{\cos x}$

⑨ $\lim_{x \to \pi/2} \frac{1 + \sin x}{\cos^2 x}$

⑩ $\lim_{x \to 0^+} \frac{\ln x}{\cot x}$

⑪ $\lim_{x \to 0} \frac{x\cos x + e^{-x}}{x^2}$

* ⑫ $\lim_{x \to \infty} \frac{2x^2 + 3x + 1}{5x^2 + x + 4}$

⑬ $\lim_{x \to 0} \frac{\sin^{-1}(2x)}{\sin^{-1}(x)}$

⑭ $\lim_{x \to -\infty} \frac{3 - 3^x}{5 - 5^x}$

⑮ $\lim_{x \to 1} \frac{2x^3 - 5x^2 + 6x - 3}{x^3 - 2x^2 + x - 1}$

⑯ $\lim_{x \to 0} \frac{x - \tan^{-1} x}{x \sin x}$

* can do without L'Hopital's Rule

BC: Q303 CH8 Lesson 1 Notes (Please Bring Additional Paper to Take Notes**)**

BC: Q303 CH8 Lesson 1 HW:

Textbook Section 8.2: 5, 13, 17, 21, 25, 31, 33-51 odd

BC: Q303 CH8. Lesson 2

I. Definition: Improper Integrals with Infinite Integration Limits (PG: 459)

Ex A: Evaluate $\int_0^\infty e^{-2x}\,dx$

Ex B: Evaluate $\int_0^\infty x^2\,dx$

Ex C: Evaluate $\int_{-\infty}^1 e^x\,dx$

Ex D: Evaluate $\int_{-\infty}^\infty \frac{1}{1+x^2}\,dx$

II. Definition: Improper Integrals with Infinite Discontinuities (PG: 463)

Example E: Evaluate $\int_0^3 \frac{dx}{(x-1)^{\frac{2}{3}}}$

Example F: Evaluate $\int_0^4 \frac{dx}{(x-3)^2}$

Example G: Evaluate $\int_0^1 \frac{dx}{\sqrt{x}}$

III. Lets investigate $\int_1^{\infty} \frac{1}{x^p}\, dx$.

A. Find $\int_1^{\infty} \frac{1}{\boldsymbol{x}^1}\, \boldsymbol{dx}$

B. Find $\int_1^{\infty} \frac{1}{\boldsymbol{x}^2}\, \boldsymbol{dx}$

C. Find $\int_1^{\infty} \frac{1}{\boldsymbol{x}^{\left(\frac{1}{2}\right)}}\, \boldsymbol{dx}$

Conclusions:

IV. Direct (or Basic) Comparison Test (Pg. 464)

A. Prove $\int_{1}^{\infty} \frac{dx}{\sqrt{x^2 - 0.01}}$ converges or diverges by the DCT.

B. Prove $\int_{1}^{\infty} \frac{\sin^2 x}{x^2}$ converges or diverges by the DCT.

BC.Q303 CH8 Lesson 2 Homework (Section 8.4):

2. Consider $\int_1^{\infty} \frac{dx}{x^{\frac{1}{3}}}$

A. Can you determine by observation whether or not the integral converges or diverges? If you can determine by observation, state whether it converges or diverges. If you cannot determine by observation, state that further investigation is required.
B. Rewrite the integral using a limit definition.
C. If you determined from part A that either (1) the integral converges or (2) that it needed further investigation, evaluate the integral's value ---- otherwise ignore part C.

11. Consider $\int_{-\infty}^{-2} \frac{2dx}{x^2 - 1}$

A. Can you determine by observation whether or not the integral converges or diverges? If you can determine by observation, state whether it converges or diverges. If you cannot determine by observation, state that further investigation is required.
B. Rewrite the integral using a limit definition.
C. If you determined from part A that either (1) the integral converges or (2) that it needed further investigation, evaluate the integral's value ---- otherwise ignore part C.
NOTE: You should employ partial fractions when evaluating this integral.

21. Consider $\int_{-\infty}^{\infty} e^{-|x|} dx$

A. Can you determine by observation whether or not the integral converges or diverges? If you can determine by observation, state whether it converges or diverges. If you cannot determine by observation, state that further investigation is required.
B. Rewrite the integral using a limit definition.
C. If you determined from part A that either (1) the integral converges or (2) that it needed further investigation, evaluate the integral's value ---- otherwise ignore part C.
NOTE: Use the definition of absolute value to make turn the integrand into a piecewise expression. Then recognize the symmetric nature of the integrand.

25. Consider $\int_0^2 \frac{dx}{1 - x^2}$

A. Can you determine by observation whether or not the integral converges or diverges? If you can determine by observation, state whether it converges or diverges. If you cannot determine by observation, state that further investigation is required.
B. Rewrite the integral using a limit definition.
C. If you determined from part A that either (1) the integral converges or (2) that it needed further investigation, evaluate the integral's value ---- otherwise ignore part C.
NOTE: You should employ partial fractions when evaluating this integral.

35. Consider $\int_{0}^{\ln 2} y^{-2} e^{\frac{1}{y}} dy$

A. Can you determine by observation whether or not the integral converges or diverges? If you can determine by observation, state whether it converges or diverges. If you cannot determine by observation, state that further investigation is required.
B. Rewrite the integral using a limit definition.
C. If you determined from part A that either (1) the integral converges or (2) that it needed further investigation, evaluate the integral's value ---- otherwise ignore part C.
NOTE: You should employ a u-substitution when evaluating this integral.

40. Consider $\int_{-\infty}^{0} xe^{x} dx$

A. Can you determine by observation whether or not the integral converges or diverges? If you can determine by observation, state whether it converges or diverges. If you cannot determine by observation, state that further investigation is required.
B. Rewrite the integral using a limit definition.
C. If you determined from part A that either (1) the integral converges or (2) that it needed further investigation, evaluate the integral's value ---- otherwise ignore part C.
NOTE: You should employ integration by parts and L'Hopital's rule when evaluating this integral.

43. Find the area of the region in the first quadrant that lies under the curve $y = \frac{\ln x}{x^2}$.
NOTE: You should employ integration by parts and L'Hopital's rule when evaluating this integral.

Use the **Direct Comparison Test** to prove whether or not the following integrals converge or diverge.

32. $\int_{1}^{\infty} \frac{dx}{x^3 + 1}$

33. $\int_{\pi}^{\infty} \frac{2 + \cos x}{x} dx$

SUPP: $\int_{2}^{\infty} \frac{dx}{\sqrt{x^2 - 1}}$

BC: Q303 CH8. Lesson 3

I. Let's investigate: $\int_a^{\infty} \frac{1}{e^x}\,dx$ for the constant $a > 0$

II. Limit Comparison Test

A. Prove $\int_1^\infty \frac{dx}{1+x^2}$ converges or diverges by the LCT.

B. Prove $\int_3^\infty \frac{dx}{e^x - 5}$ converges or diverges using the LCT.

C. Explain why you cannot determine whether $\int_3^\infty \frac{dx}{e^x - 5}$ converges or diverges using the DCT.

III. MISC PROOFS

A. Prove $\int_1^\infty e^{-x^2}\,dx$ converges or diverges.

B. Prove $\int_2^\infty \frac{dx}{\ln x}$ converges or diverges.

C. Find if $\int_{1}^{\infty} \frac{dx}{1+x^2}$ converges or diverges. If it converges, find what it converges to.