Calculus BC: Q203 – Lesson 1: Integration by Parts

Up to this stage we have been unable to evaluate integrals such as the following:

$\int \ln x dx, \quad \int xe^x dx, \quad \int x^2 \sin x dx, \quad \int \tan^{-1} x dx$

The next formula will enable us to evaluate not only these, but also many other types of integrals.

If $u = f(x)$ and $v = g(x)$ and if f' and g' are continuous, then $\int udv = uv - \int vdu$.

Proof:

Example 1: Evaluate $\int xe^{2x} dx$.

Example 2: Evaluate $\int x \sec^2 x dx$

Example 3: Evaluate $\int \ln x dx$

Example 4: Evaluate $\int x^2 e^{2x} dx$.

Example 5: Evaluate $\int e^x \cos x dx$.

Evaluate $\int \sec^3 x dx$.

Tabular integration

(Q203 – Lesson 1) **Integration by Parts** Supplement Homework

Evaluate each integral.

1. $\int xe^{-x}dx$

2. $\int x\sec x\tan xdx$

3. $\int \cot^{-1} xdx$

4. $\int e^{-x}\sin xdx$

5. $\int \sin x\ln(\cos x)dx$

6. $\int \csc^3 xdx$

7. $\int e^{4x}\sin 5xdx$

8. $\int \cos\sqrt{x}dx$

Calculus BC: Q203 – Lesson 2 (Section 6.5 Notes): Integration by Partial Fractions

Calculus BC: Q203 – Lesson 2 (Supplemental Notes): Integration by Trigonometric Substitutions

Expression in Integrand	Trigonometric Substitution
$a^2 - x^2$	$x = a\sin\theta$
$a^2 + x^2$	$x = a\tan\theta$
$x^2 - a^2$	$x = a\sec\theta$

Example 1: Evaluate $\int \dfrac{dx}{x^2\sqrt{16 - x^2}}$

Example 2: Evaluate $\int \frac{dx}{\sqrt{4+x^2}}$

Example 3: Evaluate $\int \frac{\sqrt{x^2-9}dx}{x}$

HW: SUPPLEMENT – Evaluate Each

#42. $\int \frac{8dx}{x^2\sqrt{4-x^2}}$

#35. $\int \frac{dx}{\sqrt{9+x^2}}$

#37. $\int \frac{dx}{\sqrt{4x^2-49}}$

#39. $\int \frac{x^3 dx}{\sqrt{1-x^2}}$

MISC. EXTRA: $\int \sin^3 x \, dx = \int \sin^2 x \cdot \sin x \, dx = \int \left(1 - \cos^2 x\right) \cdot \sin x \, dx$

Finish using $u = \cos x$ substitution.