

Calculus BC: Q203 – Lesson 1: Integration by Parts

Up to this stage we have been unable to evaluate integrals such as the following:

$$\int \ln x dx, \quad \int x e^x dx, \quad \int x^2 \sin x dx, \quad \int \tan^{-1} x dx$$

The next formula will enable us to evaluate not only these, but also many other types of integrals.

If $u = f(x)$ and $v = g(x)$ and if f' and g' are continuous, then $\int u dv = uv - \int v du$.

Proof:

Example 1: Evaluate $\int x e^{2x} dx$.

Example 2: Evaluate $\int x \sec^2 x dx$

Example 3: Evaluate $\int \ln x dx$

Example 4: Evaluate $\int x^2 e^{2x} dx$.

Example 5: Evaluate $\int e^x \cos x dx$.

Evaluate $\int \sec^3 x dx$.

Tabular integration

(Q203 – Lesson 1) **Integration by Parts** Supplement Homework

Evaluate each integral.

1. $\int x e^{-x} dx$

2. $\int x \sec x \tan x dx$

3. $\int \cot^{-1} x dx$

4. $\int e^{-x} \sin x dx$

5. $\int \sin x \ln(\cos x) dx$

6. $\int \csc^3 x dx$

7. $\int e^{4x} \sin 5x dx$

8. $\int \cos \sqrt{x} dx$

Calculus BC: Q203 – Lesson 2 (Section 6.5 Notes): Integration by Partial Fractions

Calculus BC: Q203 – Lesson 2 (Supplemental Notes): Integration by Trigonometric Substitutions

Expression in Integrand	Trigonometric Substitution
$a^2 - x^2$	$x = a \sin \theta$
$a^2 + x^2$	$x = a \tan \theta$
$x^2 - a^2$	$x = a \sec \theta$

Example 1: Evaluate $\int \frac{dx}{x^2 \sqrt{16 - x^2}}$

Example 2: Evaluate $\int \frac{dx}{\sqrt{4+x^2}}$

Example 3: Evaluate $\int \frac{\sqrt{x^2 - 9} dx}{x}$

HW: SUPPLEMENT – Evaluate Each

#42. $\int \frac{8dx}{x^2 \sqrt{4-x^2}}$

#35. $\int \frac{dx}{\sqrt{9+x^2}}$

#37. $\int \frac{dx}{\sqrt{4x^2-49}}$

#39. $\int \frac{x^3 dx}{\sqrt{1-x^2}}$

MISC. EXTRA: $\int \sin^3 x \, dx = \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx$
Finish using $u = \cos x$ substitution.

Review : Q203 CHAPTER 6.3, 6.5, TRIG SUB

① Evaluate $\int \frac{dx}{1+9x^2}$ using A] u-substitution / Book memories
B] TRIG SUB

② Evaluate $\int \frac{dx}{\sqrt{1-x^2}}$ using A] BOOK OF memories
B] TRIG SUB

③ Evaluate $\int \frac{dx}{1-x^2}$ using A] PARTIAL FRACTIONS
B] TRIG SUB

④ Evaluate $\int \frac{(\ln x)^2}{x^3} dx$ [You choose]

⑤ Evaluate $\int \sin(\ln x) dx$ [You choose]

⑥ Evaluate $\int \frac{\sqrt{x^2-9}}{x^3} dx$ [You choose]

⑦ Evaluate $\int x 2^x dx$ [You choose]