BC – Q202: ANTIDERIVATIVES LESSON 1

Definition: A function *F* is an antiderivative of the function *f* on the interval I if $F'(x) = f(x)$ for every *x* in I.

Definition: The notation $\int f(x)dx = F(x) + C$ where $F'(x) = f(x)$ and C is an arbitrary constant, denotes the family of all antiderivatives of *f*(*x*) on an interval I.

BOOK OF MEMORIES

Examples:

To solve a differential equation is to find the original function $y = f(x)$ where $\frac{dy}{dx} = f'(x)$ is given along with some condition so that you can solve for the constant C.

1. Solve the differential equation $\frac{dy}{dx} = 6x^2 + x - 5$ subject to the condition $y = 10$ when $x = 2$.

2. Solve the differential equation $f''(x) = 5\cos x + 2\sin x$ subject to the initial conditions $f(0) = 3$ and $f'(0) = 4$.

3. Solve the differential equation $\frac{dy}{dx} = 7x^3 + \frac{1}{x^3} + 5x$ subject to the condition $y(1) = 3$.

4. Solve the differential equation $f'(x) = 5e^x + \frac{1}{1+x^2} + 4$ subject to the condition $f(0) = 6$.

AB @301 LESSON 1 HW (BC @202)

FROM TEXT SECTION 4.2

29 – 34: FIND ALL POSSIBLE FUNCTIONS f WITH GIVEN DERIVATIVE

29. $f'(x) = x$
30. $f'(x) = 2$
31. $f'(x) = 3x^2 - 2x + 1$
32. $f'(x) = \sin x$
33. $f'(x) = e^x$
34. $f'(x) = \frac{1}{x-1}$; $x > 1$

35 – 38: FIND THE FUNCTION WITH THE GIVEN DERIVATIVE WHOSE GRAPH PASSES THROUGH THE POINT P.

35. $f'(x) = -\frac{1}{x^2}$; $x > 0$ $P(2, 1)$
36. $f'(x) = \frac{1}{4x^{3/4}}$ $P(1, -2)$
37. $f'(x) = \frac{1}{x+2}$; $x > -2$ $P(-1, 3)$
38. $f'(x) = 2x + 1 - \cos x$ $P(0, 3)$

FROM TEXT SECTION 6.1

FIND THE GENERAL SOLUTION TO THE DIFFERENTIAL EQUATION

1. $\frac{dy}{dx} = 5x^4 - \sec^2 x$
2. $\frac{dy}{dx} = \sec x \tan x - e^x$
3. $\frac{dy}{dx} = \sin x - e^{-x} + 8x^3$
4. $\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2}$
5. $\frac{dy}{dx} = 5^x \ln 5 + \frac{1}{1+x^2}$
6. $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{x}}$

CONTINUED ...

FIND THE PARTICULAR SOLUTION TO THE DIFFERENTIAL EQUATION

11. $\frac{dy}{dx} = 3\sin x$; $y = 2$ when $x = 0$

12. $\frac{dy}{dx} = 2e^x - \cos x$; $y = 3$ when $x = 0$

13. $\frac{dy}{dx} = 7x^6 - 3x^2 + 5$; $y = 1$ when $x = 1$

14. $\frac{dy}{dx} = 10x^9 + 5x^4 - 2x + 4$; $y = 6$ when $x = 1$

15. $\frac{dy}{dx} = -\frac{1}{x^2} - \frac{3}{x^4} + 12$; $y = 3$ when $x = 1$

16. $\frac{dy}{dx} = 5\sec^2 x - \frac{3}{2}\sqrt{x}$; $y = 7$ when $x = 0$

17. $\frac{dy}{dt} = \frac{1}{1+t^2} + 2^t \ln 2$; $y = 3$ when $t = 0$

18. $\frac{dx}{dt} = \frac{1}{t} - \frac{1}{t^2} + 6$; $x = 0$ when $t = 1$

19. $\frac{dv}{dt} = 4\sec(t)\tan(t) + e^t + 6t$; $v = 5$ when $t = 0$

20. $\frac{ds}{dt} = t(3t - 2)$; $s = 0$ when $t = 1$

BC – Q202: ANTIDERIVATIVES LESSON 2 (Method of U - Substitution)

If F is an antiderivative of f, then $\int f(g(x))g'(x)dx = F(g(x)) + C$

If $u = g(x)$ and $du = g'(x)dx$ then $\int f(u)du = F(u) + C$

1. Evaluate $\int \sqrt{5x+7}dx$

2. Evaluate $\int \cos(4x)\,dx$

3. Evaluate $\int (2x^3 + 1)^7 x^2 dx$

4. Evaluate $\int x\sqrt{7-6x^2}\,dx$

5. Evaluate $\int \frac{x^2-1}{\left(x^3-3x+1\right)^6}\,dx$

6. Evaluate $\int \cos^3(5x)\sin(5x)dx$

7. Evaluate $\int \frac{x}{3x^2 - 5} dx$

8. Evaluate $\int \frac{1}{9 - 2x} dx$

9. Evaluate $\int \frac{1}{(9 - 2x)^3} dx$

10. Evaluate $\int \frac{e^{3/x}}{x^2}\,dx$

11. Evaluate $\int x(x-3)^2\,dx$

12. Evaluate $\int \frac{\sqrt{\ln x}}{x}\,dx$

13. Evaluate $\int \tan x dx$

14. Evaluate $\frac{d}{dx}\int 5\cos(e^{2x})dx$

15. Evaluate $\int \sec x dx$

"MINI U-SUBS"

16. Evaluate $\frac{dy}{dx} = 3e^{2x} + 6e^{-3x} + \cos\left(\frac{x}{4}\right)$ if $y = 4$ when $x = 0$.

17. Evaluate $f''(x) = 3e^{-x}$ with conditions: $f(0) = -1$ and $f'(0) = 1$

18. Evaluate $\int \frac{1}{1+4x^2}\,dx$

19. Evaluate $\int \frac{\left(e^x+1\right)^2}{e^x}\,dx$

HW Problems

1. $\int x^2 \sqrt[3]{3x^3+7}\,dx$

2. $\int \frac{\left(1+\sqrt{x}\right)^3}{\sqrt{x}}dx$

3. $\int n^2\sqrt{n^3-1}dn$

4. $\int \cos(3x)\sqrt[3]{\sin(3x)}dx$

5. $\int \frac{\cos t}{(1-\sin t)^2}dt$

6. $\int \sec^2(3x)\tan(3x)\,dx$

7. $\int x\cot\left(x^2\right)\csc\left(x^2\right)dx$

8. Solve the differential equation: $f'(x)=\sqrt[3]{3x+2};\quad f(2)=9$

9. Solve the differential equation: $\frac{dy}{dx}=x\sqrt{x^2+5};\quad y=12 \text{ if } x=2$

10. $\int \frac{1}{2x+7}dx$

11. $\int \frac{4x}{x^2-9}dx$

12. $\int e^{-4x}dx$

13. $\int \tan(2x)dx$

14. $\int \frac{x-2}{x^2-4x+9}dx$

15. $\int \frac{\ln x}{x}dx$

16. $\int \frac{7}{6x\sqrt{36x^2-1}}$ for $x>0$.