PRACTICE REVIEW # 1 Solution

$$\int \frac{x}{\sqrt[5]{3x+2}}\,dx \qquad u = \sqrt[5]{3x+2} \qquad u^5 = 3x+2 \qquad 5u^4\,du = 3\,dx$$

$$\frac{u^5-2}{3} = x \qquad \frac{5}{3}u^4\,du = dx$$

$$= \int \frac{\frac{u^5-2}{3}\cdot\frac{5}{3}u^4}{u}\,du = \frac{5}{9}\int (u^8 - 2u^3)\,du$$

$$= \frac{5}{9}\frac{u^9}{9} - \frac{10}{9}\frac{u^4}{4} + C$$

$$= \frac{5}{81}(3x+2)^{9/5} - \frac{10}{36}(3x+2)^{4/5} + C$$

$$= \boxed{\frac{5}{81}(3x+2)^{9/5} - \frac{5}{18}(3x+2)^{4/5} + C}$$

$$= \frac{10}{162}(3x+2)^{9/5} - \frac{45}{162}(3x+2)^{4/5} + C$$

$$\frac{5}{162}(3x+2)^{4/5}\left(\underbrace{2(3x+2) - 9}_{6x+4-9}\right) + C$$

$$\frac{5}{162}(3x+2)^{4/5}(6x-9) + C$$

Practice Review # 2 Solution

$$\int \frac{dx}{1+\sin x+\cos x} = \int \frac{1}{\left[1+\left(\frac{2u}{1+u^2}\right)+\left(\frac{1-u^2}{1+u^2}\right)\right]}\cdot\frac{2}{1+u^2}\,du$$

$$= 2\int \frac{du}{(1+u^2)+(2u)+(1-u^2)}$$

$$= 2\int \frac{du}{2+2u} = \int \frac{du}{1+u} = \ln|1+u|+C$$

$$= \ln\left|1+\tan\left(\tfrac{x}{2}\right)\right|+C$$

Practice Review #3 Solution

$$\int \frac{dx}{\sqrt{7+6x-x^2}} = \int \frac{dx}{\sqrt{(-1)(x^2-6x-7)}} = \int \frac{dx}{\sqrt{(-1)(x^2-6x+9-16)}}$$

$$= \int \frac{dx}{\sqrt{(-1)((x-3)^2-16)}} = \int \frac{du}{\sqrt{16-u^2}} \ominus \frac{1}{4}\int \frac{du}{\sqrt{1-\left(\frac{u}{4}\right)^2}}$$

$$= \frac{4}{4}\sin^{-1}\left(\frac{u}{4}\right)+C$$

OR

$$= \sin^{-1}\left(\frac{x-3}{4}\right)+C$$

Let $u = 4\sin\theta$

$du = 4\cos\theta\, d\theta$

$$\ominus \int \frac{4\cos\theta\, d\theta}{\sqrt{16-16\sin^2\theta}} = \int \frac{\cos\theta}{\cos\theta}\,d\theta = \int d\theta = \theta + C$$

$$= \sin^{-1}\left(\frac{u}{4}\right)+C$$

$$= \sin^{-1}\left(\frac{x-3}{4}\right)+C$$

[redacted] PRACTICE REVIEW #4 Solution

$$\int \frac{dx}{\tan x + \sin x} = \int \frac{1}{\left[\left(\frac{2u}{1-u^2}\right) + \left(\frac{2u}{1+u^2}\right)\right]} \frac{2\,du}{(1+u^2)}$$

$$= 2\int \frac{du}{\frac{(2u)(1+u^2)}{1-u^2} + \frac{2u(1-u^2)}{1-u^2}} = 2\int \frac{du}{\frac{2u + \cancel{2u^3} + 2u - \cancel{2u^3}}{1-u^2}}$$

$$= 2\int \frac{1-u^2}{4u} = \frac{1}{2}\int \frac{1-u^2}{u}\,du = \frac{1}{2}\int (u^{-1} - u)\,du$$

$$= \frac{1}{2}\ln|u| - \frac{1}{4}u^2 + c = \frac{1}{2}\ln\left|\tan\left(\frac{x}{2}\right)\right| - \frac{1}{4}\tan^2\left(\frac{x}{2}\right) + c$$