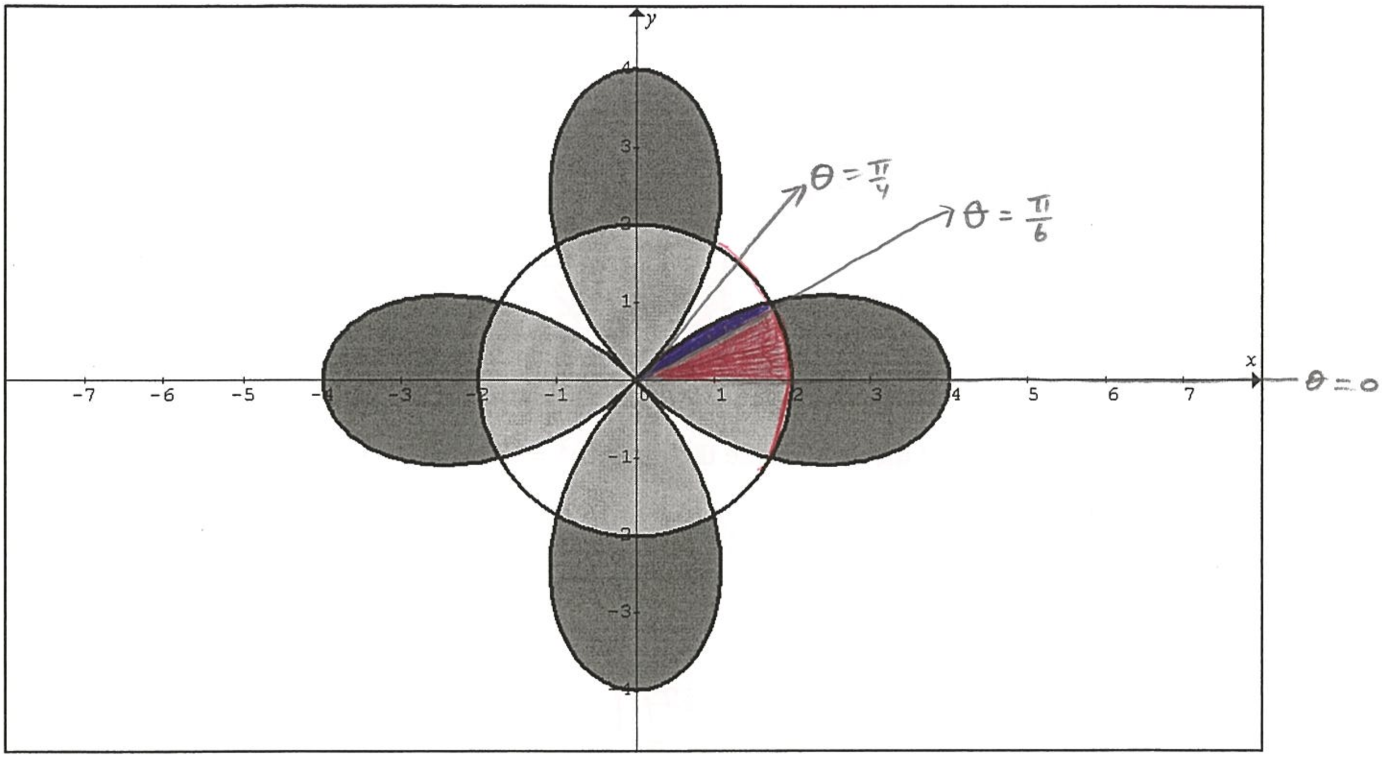
BC: Q403 CHAPTER 10 – LESSON 4 (REVIEW)



1. The diagram above shows the graphs of $r = 4\cos(2\theta)$ and $r = 2$. Set up, but do not evaluate, an expression involving one or more integrals, used to find the area of the light shaded region.

$4\cos(2\theta) = 2 \qquad \cos(2\theta) = \frac{1}{2} \qquad 2\theta = \frac{\pi}{3} \rightarrow \theta = \frac{\pi}{6}$

$4\cos(2\theta) = 0 \qquad \cos(2\theta) = 0 \qquad 2\theta = \frac{\pi}{4} \rightarrow \theta = \pi/4$

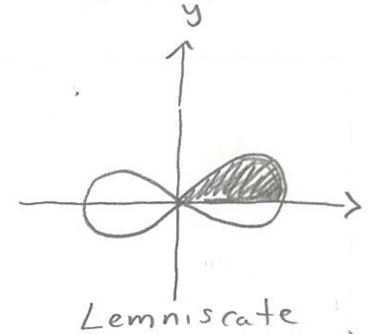
$$A = 8\int_0^{\pi/6} \frac{1}{2}(2)^2\,d\theta + 8\int_{\pi/6}^{\pi/4} \frac{1}{2}(4\cos(2\theta))^2\,d\theta$$

9.827

2. Revisit HW #47 Find the area within one loop of $r^2 = 4\cos(2\theta)$

$r^2 = 4\cos 2\theta \qquad r = +\sqrt{4\cos 2\theta}$

θ	0	$\pi/4$	$\pi/2$	$\frac{3\pi}{4}$	π
r	2	0	DNE	0	2



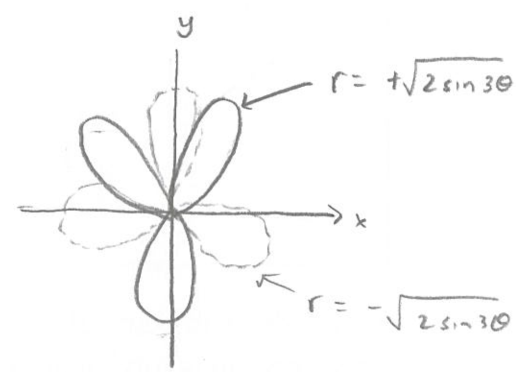
Lemniscate
(infinity symbol)

$$A = 2\int_0^{\pi/4} \frac{1}{2}(4\cos 2\theta)\,d\theta = \boxed{2}$$

$r = \sqrt{2\sin(\frac{\pi}{2})}$

3. Text Problem #48. Find the area inside the curve $r^2 = 2\sin(3\theta)$. $\rightarrow r = \pm\sqrt{2\sin 3\theta}$

θ	0	$\pi/6$	$\pi/3$	$\leftarrow \pi/2 \rightarrow$	$\frac{2\pi}{3}$	$5\pi/6$	π
r	0	$\sqrt{2}$	0	DNE	0	$\sqrt{2}$	0



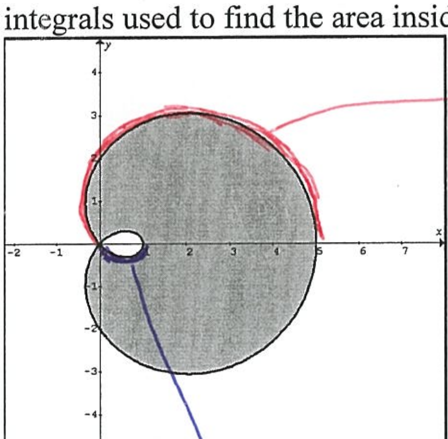
6-petal rose

$$A = 6\int_0^{\pi/3} \frac{1}{2}(2\sin 3\theta)\,d\theta = \boxed{4}$$

$0 \le 3\theta \le \pi \qquad 0 \le \theta \le \frac{\pi}{3} \qquad \pi < 3\theta \le 2\pi$

$\pi/3 < \theta < 2\pi/3$

4. Consider $r = 3\cos(\theta) + 2$. Set up, but do not evaluate, an expression involving one or more integrals used to find the area inside the large loop but outside the small loop. Limacon



$0 \le \theta \le \cos^{-1}(-2/3)$

$r = 0: \quad 3\cos\theta + 2 = 0$

$3\cos\theta = -2$

$\cos\theta = -2/3$

$\theta = \cos^{-1}(-2/3)$

$y = 0: \quad r\sin\theta = 0 \rightarrow (3\cos\theta + 2)\sin\theta = 0$

$\theta = \cos^{-1}(-2/3), \ \theta = 0, \pi$

$\cos^{-1}(-2/3) \le \theta \le \pi$

$$A = 2\int_0^{\cos^{-1}(-2/3)} \frac{1}{2}(3\cos\theta + 2)^2\,d\theta - 2\int_{\cos^{-1}(-2/3)}^{\pi} \frac{1}{2}(3\cos\theta + 2)^2\,d\theta$$

Review Solutions

1a. $\dfrac{dy}{dx}=\dfrac{dy/dt}{dx/dt}=\dfrac{2e^{2t}}{2t}=\dfrac{e^{2t}}{t}=y'$

1b. $\dfrac{d^2y}{dx^2}=\dfrac{d(y')}{dx}=\dfrac{d(y')}{1}\cdot\dfrac{1}{dx}=\left(\dfrac{2te^{2t}-e^{2t}}{t^2}\right)\cdot\left(\dfrac{1}{2t}\right)=\left(\dfrac{2te^{2t}-e^{2t}}{2t^3}\right)=e^{2t}\left(\dfrac{2t-1}{2t^3}\right)$

2. $L=\int_1^5\sqrt{\left(\dfrac{dy}{dt}\right)^2+\left(\dfrac{dx}{dt}\right)^2}\,dt=\int_1^5\sqrt{(2t)^2+(4t+1)}dt=\int_1^5\sqrt{(2t+1)^2}\,dt=\int_1^5(2t+1)dt=28$

3. $S=\int_5^9 2\pi(radius)\sqrt{\left(\dfrac{dy}{dt}\right)^2+\left(\dfrac{dx}{dt}\right)^2}=\int_5^9 2\pi(12t)\sqrt{(t)^2+(12)^2}\,dt=9424\pi$

7b. $v(t)=\left[\dfrac{1}{(t+2)}\right]i+(2t)j$ $\qquad a(t)=\left[\dfrac{-1}{(t+2)^2}\right]i+2j$

8. $r=(e^{3t}+c_1)i+(t^2+c_2)j\Rightarrow r(0)=(1+c_1)i+(c_2)j=i-4j\Rightarrow c_1=0,c_2=-4$

$r=(e^{3t})i+(t^2-4)j$

13. (E)

14. $r=\dfrac{1}{\cos^2\theta}\Rightarrow r\cos^2\theta=1\Rightarrow r^2\cos^2\theta=r\Rightarrow x^2=\sqrt{x^2+y^2}$

15.

$$\frac{dy}{dx}=\frac{f(\theta)\cos\theta+f'\sin\theta}{f'(\theta)\cos\theta-f(\theta)\sin\theta}=\left.\frac{(-2\cos3\theta)\cos\theta+(6\sin3\theta)\sin\theta}{(6\sin3\theta)\cos\theta-(-2\cos3\theta)\sin\theta}\right|_{\theta=\pi/6}=\frac{6\sin(\pi/6)}{6\cos(\pi/6)}=\frac{\sqrt{3}}{3}=0.57735$$

16. $0\le\theta\le2\pi$ to complete one revolution.

$$A=\int_0^{2\pi}\left(\frac{1}{2}(5-2\cos\theta)^2\,d\theta\right)=27\pi$$

17. $L=\displaystyle\int_0^{\pi/3}\sqrt{\left[\left(5\sin^2\left(\frac{\theta}{2}\right)\right)^2+\left(5\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right)^2\right]}d\theta=$

$$\int_0^{\frac{\pi}{3}}\sqrt{\left(25\sin^2\frac{\theta}{2}\right)\left(\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}\right)}d\theta=\int\left(5\sin\frac{\theta}{2}\right)d\theta=-10\cos\left.\frac{\theta}{2}\right|_0^{\pi/3}=10-5\sqrt{3}$$

9. $x(3.1)=1+\int_1^{3.1}\ln\sqrt{t}\,dt\approx1.704$ $\qquad(1.704,\ 2.221)$

$y(3.1)=2+\int_1^{3.1}-\cos(e^t)\,dt\approx2.221$