BC. Q402. CH9B. PRACTICE EXAM Solutions

X. a) $\dfrac{f'''(2)(x-2)^3}{3!} = -\dfrac{4}{3}(x-2)^3$

$\therefore \dfrac{f'''(2)}{3!} = -\dfrac{4}{3} \quad \therefore f'''(2) = -4\cdot 2 = -8$

b) $f(1.5) \approx P_3(1.5) = -3 + 5(-0.5) + \frac{3}{2}(-0.5)^2 - \frac{4}{3}(-0.5)^3$
$= -3 - \frac{5}{2} + \frac{3}{8} + \frac{1}{6} \approx -4.95833$

c) $\left|R_3(1.5)\right| = \dfrac{f''''(c)(-0.5)^4}{4!} \le \dfrac{3(-0.5)^4}{4!} < 0.00782$

$-4.95833 \pm 0.00782 \quad > -5$

Y. a) $\displaystyle\lim_{x\to 0} \frac{f(x)-\frac{1}{3}}{x} = \lim_{x\to 0} \frac{\frac{2}{3^2}x + \frac{3}{3^3}x + \cdots + \frac{(n+1)}{3^{(n+1)}}x^{n-1} + \cdots}{x}$

$= \displaystyle\lim_{x\to 0}\left[\frac{2}{3^2} + \cancel{\frac{3}{3^3}x} + \cdots + \cancel{\frac{(n+1)x^{n-1}}{3^{(n+1)}}} + \cdots\right]$

$= \dfrac{2}{9}$

b) $\displaystyle\int_0^1 f(x)\,dx = \int_0^1 \left(\frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots\right)dx$

$= \left.\frac{1}{3}x + \frac{x^2}{3^2} + \frac{x^3}{3^3} + \cdots \right]_0^1$

$= \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + \cdots$ which is geometric
$a = \frac{1}{3} \quad r = \frac{1}{3}$

c) $S = \dfrac{\frac{1}{3}}{1-\frac{1}{3}} = \dfrac{\frac{1}{3}}{\frac{2}{3}} = \dfrac{1}{2}$

7. $f(x) = e^{-2x^2}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

a) $$e^{-2x^2} = 1 + (-2x^2) + \frac{(-2x^2)^2}{2!} + \frac{(-2x^2)^3}{3!} + \cdots + \frac{(-2x^2)^n}{n!} + \cdots$$

$$= 1 - 2x^2 + 2x^4 - \frac{4x^6}{3} + \cdots + \frac{(-2)^n x^{2n}}{n!} + \cdots$$

b) RATIO Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{2^{n+1}x^{2n+2}}{(n+1)!}\cdot\frac{n!}{2^n x^{2n}}\right|$$

$$= \lim_{n\to\infty}\frac{2}{n+1}\left|x^2\right| = 0 < 1$$

∴ The series will converge for all real values of x.

c) $$|f(x) - g(x)| = |\text{error}| = |R_3(x)| \le \frac{(-2x^2)^4}{4!}$$

$$\le \frac{-2^4(0.6)^8}{4!}$$

$$\approx 0.011198 < 0.02$$

This is true b/c the terms of the series $f(x)$ is strictly alternating and decreasing in absolute value to zero for all x on $[-0.6, 0.6]$

W. $f(x) = (9+x)^{-1/2}$

a) $f(0) = \frac{1}{3}$

$f'(x) = -\frac{1}{2}(9+x)^{-3/2}$

$f'(0) = -\frac{1}{54}$

$f''(x) = \frac{3}{4}(9+x)^{-5/2}$

$f''(0) = \frac{3}{4} \cdot \frac{1}{3^5}$

$f'''(x) = -\frac{15}{8}(9+x)^{-7/2}$

$f'''(0) = -\frac{15}{8} \cdot \frac{1}{3^7}$

$f(x) \approx P_2(x) = \frac{1}{3} - \frac{1}{54}x + \frac{1}{4 \cdot 3^4}\frac{x^2}{2!} = \frac{1}{3} - \frac{1}{54}x + \frac{1}{648}x^2$

b) $1/\sqrt{10} = f(1) \approx \frac{1}{3} - \frac{1}{54} + \frac{1}{8 \cdot 3^4}$

with $|\text{error}| \leq \left| \frac{-15}{8 \cdot 3^7} \frac{(1)^3}{3!} \right| = \frac{15}{104976} < \frac{15}{100001}$

* This is true b/c the series for $f(1)$ is strictly alt. and decreasing in absolute value to zero. (E1)

c) $W(x) = \int W'(x)\,dx = \int f(x)\,dx = \int \left(\frac{1}{3} - \frac{1}{54}x + \frac{x^2}{648} - \cdots \right) dx$

$= \left(\frac{1}{3}x - \frac{x^2}{108} + \cdots \right) + C$

$W(0) = \left(\frac{1}{3}(0) - \frac{0}{108} + \cdots \right) + C = -4/9 \quad \therefore \quad C = -4/9$

$W(x) \approx -\frac{4}{9} + \frac{1}{3}x - \frac{x^2}{108}$