BC.Q402.Lesson4.Practice Exam

X. [15] The function f has derivatives of all orders for all real numbers x. The third-degree Taylor polynomial for f about $x = 2$ is $P_3(x) = -3 + 5(x-2) + \frac{3}{2}(x-2)^2 - \frac{4}{3}(x-2)^3$.

(a) Find $f'''(2)$.

(b) Approximate $f(1.5)$ using the third-degree polynomial.

(c) The fourth derivative of f satisfies the inequality $|f^4(x)| \leq 3$ for all x in the closed interval [1.5, 2]. Use the Lagrange error bound (Remainder Estimation Theorem) on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \neq -5$.

Y. [24] A function f is defined by $f(x) = \frac{1}{3} + \frac{2}{3^2}x + \frac{3}{3^3}x^2 + \cdots + \frac{n+1}{3^{n+1}}x^n + \cdots$ for all x in the interval of convergence of the given series.

(a) Find $\lim_{x \to 0} \dfrac{f(x) - \frac{1}{3}}{x}$

(b) Write the first three nonzero terms and the general term for an infinite series that represents $\int_0^1 f(x)dx$.

(c) Find the sum of the series determined in part (b).

Z [28] Let f be a function given by $f(x) = e^{-2x^2}$

(a) find the first four nonzero terms and the general term of the power series for $f(x)$ about $x = 0$

(b) Find the interval of convergence of the power series for f(x) about $x = 0$. Show the analysis that leads to your conclusion.

(c) Let g be a function given by the first four nonzero terms of the power series for $f(x)$ about $x = 0$. Show that $|f(x) - g(x)| < 0.02$ for $-0.6 \leq x \leq 0.6$

W. Let f be the function defined by $f(x) = \dfrac{1}{\sqrt{9+x}}$.

(a) Construct a second-order Taylor Polynomial for $f(x)$ about $x = 0$.

(b) Show that if the polynomial in part (a) were used to approximate $1/\sqrt{10}$, then the |error| in approximation is less than $\dfrac{15}{100001}$.

(c) Suppose $w'(x) = f(x)$ where $w(0) = -\frac{4}{9}$. Write a second degree Taylor Polynomial centered about $x = 0$ for $w(x)$.