Lesson 4 Review Solutions

1. Geometric: $r = (-(4x+1)) = -4x-1$

Converges for $|-4x-1| < 1 \rightarrow |4x+1| < 1$

$-1 < 4x+1 < 1$

$-2 < 4x < 0$

$-\frac{1}{2} < x < 0$ Do Not check endpoints

OR

♡ RATIO TEST : $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \ldots = |4x+1| < 1$

$-1/2 < x < 0$ But now you do have to check endpoints

Test Endpoints | $x = -1/2$: $\sum_0^\infty (-1)^n(-1)^n = \sum_0^\infty 1 = 1 + 1 + 1 + \cdots$ Diverges

$x = 0$: $\sum_0^\infty (-1)^n = -1 + 1 - 1 + 1 \ldots$ Diverges (nth term does not equal zero)

2. ♡ RATIO TEST ... $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \ldots = \frac{1}{5}|x+3| < 1$

$|x+3| < 5 \qquad -8 < x < 2$

$x = 2$: $\sum_0^n n$ Diverges ; fails nth term test.

$x = -8$: $\sum (-1)^n n$ Diverges ; fails nth term test

$\boxed{-8 < x < 2}$

Center $x = -3$

radius $\rho = 5$

3. ♡ RATIO TEST ... $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(2^{n+1})(n+2)(x-1)^{n+1}}{2^n(n+1)(x-1)^n}\right| = 2|x-1|$

$2|x-1| < 1 \qquad |x-1| < 1/2$

$-1/2 < x-1 < 1/2$

Test Endpoints | $1/2 < x < 3/2$

$x = 1/2$: $\sum (-2)^n(n+1)(-\frac{1}{2})^n = \sum (n+1)$ diverges fails nth term test

$x = 3/2$: $\sum (-2)^n(n+1)(\frac{1}{2})^n = \sum (-1)^n(n+1)$ diverges fails nth term test

$\boxed{1/2 < x < 3/2}$

Center $x = 1$

radius $\rho = 1/2$

4. ♡ RATIO TEST ... $\lim_{n\to\infty}\left|\frac{(4x-5)^{2n+3}}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{(4x-5)^{2n+1}}\right|$

$= \lim_{n\to\infty} \frac{n^{3/2}}{(n+1)^{3/2}}\left|(4x-5)^2\right| = (4x-5)^2 < 1$

$|4x-5| < 1$

$-1 < 4x-5 < 1$

$4 < 4x < 6$

$1 < x < 3/2$

TEST ENDPOINTS

$x=1$: $\sum(-1)^{2n+1}\frac{1}{n^{3/2}} = \sum(-1)\frac{1}{n^{3/2}} = -\sum\frac{1}{n^{3/2}}$ converges by p-series $p=3/2$

$x=3/2$: $\sum\frac{1}{n^{3/2}}$ converges by p-series $p=3/2$

$$\boxed{1 \le x \le 3/2}$$

center $x = 5/4$

radius $\rho = 1/4$

5. Geometric :

$r = \left(\frac{x-2}{10}\right)$ $\cdot \left|\frac{x-2}{10}\right| < 1$ $\quad -10 < x-2 < 10$

$\boxed{-8 < x < 12}$

$S = \frac{1}{1-\left(\frac{x-2}{10}\right)}$ on $-8 < x < 12$

6. Geometric :

$r = (\ln x)$ $\quad |\ln x| < 1$ $\quad -1 < \ln x < 1$

$\boxed{e^{-1} < x < e}$

$S = \frac{1}{1-\ln x}$ on $e^{-1} < x < e$

7. $\sum_{0}^{\infty}(x)^n \approx 1+x$; $\sum_{0}^{\infty}(x)^n \approx 1+x+x^2$; $\sum_{0}^{\infty}(x)^n \approx 1+x+x^2+x^3+x^4+x^5+x^6$

for $-1<x<1$ linear quadratic six degree

8. $\sum_{0}^{\infty}(-1)^n\frac{(x-3)^n}{n+1} \approx 1 - \frac{(x-3)}{2}$ $\quad \sum_{0}^{\infty}(-1)^n\frac{(x-3)^n}{n+1} \approx 1 - \frac{(x-3)}{2} + \frac{(x-3)^2}{3}$

$\sum_{0}^{\infty}(-1)^n\frac{(x-3)^n}{n+1} \approx 1 - \frac{(x-3)}{2} + \frac{(x-3)^2}{3} - \frac{(x-3)^3}{4}$ $\quad$ for $2 < x \le 4$

Q401.CH9A. POSITIVE TERM SERIES AND CONVERGENCE/DIVERGENCE REVIEW

Determine whether each positive-term-series converges or diverges.
There may be more than one test that yields conclusive results.

1. $\sum_{n=1}^{\infty} \frac{1}{n^4+n^2+1}$ C; Direct OR LIMIT COMPARISON TEST $b_n = \frac{1}{n^4}$

2. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+4}$ D; LIMIT COMPARISON TEST $b_n = \frac{1}{n^{1/2}}$

3. $\sum_{n=1}^{\infty} \frac{1+2^n}{1+3^n}$ C; LIMIT COMPARISON TEST $b_n = \left(\frac{2}{3}\right)^n$
Very TRICKY
Think geometric

4. $\sum_{n=1}^{\infty} ne^{-n}$ C; INTEGRAL TEST

5. $\sum_{n=1}^{\infty} \frac{3n+1}{2^n}$ C; RATIO TEST

6. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$ C; RATIO TEST ; Limit Comparison $b_n = \frac{1}{n^{3/2}}$
inconclusive

7. $\sum_{n=1}^{\infty} \frac{100^n}{n!}$ C; RATIO TEST

8. $\sum_{n=1}^{\infty} \frac{1}{n^n}$ C; RATIO TEST ; ROOT TEST ; Direct $b_n = \frac{1}{n^2}$
very tricky very easy Neat

Supporting Details

1. $\frac{1}{n^4+n^2+1} < \frac{1}{n^4}$ C : DCT

$\lim\limits_{n\to\infty} \frac{n^4}{n^4+n^2+1} = 1 \in (0,\infty)$ C : LCT

2. $\lim\limits_{n\to\infty} \frac{\sqrt{n}}{n+4} \cdot \frac{\sqrt{n}}{1} = \lim\limits_{n\to\infty} \frac{n}{n+4} = 1 \in (0,\infty)$ D : LCT

3. $\lim\limits_{n\to\infty} \frac{1+2^n}{1+3^n} \cdot \frac{3^n}{2^n} = \lim\limits_{n\to\infty} \frac{3^n}{1+3^n} \cdot \frac{1+2^n}{2^n} = \lim\limits_{n\to\infty} \left(\frac{1+3^n}{3^n}\right)^{-1} \cdot \left(\frac{1+2^n}{2^n}\right)^{1}$

$= \lim\limits_{n\to\infty} \left[\left(\cancelto{0}{\frac{1}{3^n}} + 1\right)^{-1} \cdot \left(\cancelto{0}{\frac{1}{2^n}} + 1\right)\right] = 1 \in (0,\infty)$ C : LCT

4. $\int_1^\infty x e^{-x}\,dx = \lim\limits_{b\to\infty} \int_1^b x e^{-x}\,dx = \cdots = \frac{2}{e}$ C : Integral Test

5. $\lim\limits_{n\to\infty} \frac{3(n+1)+1}{2^{n+1}} \cdot \frac{2^n}{3n+1} = \frac{1}{2} \lim\limits_{n\to\infty} \frac{3(n+1)+1}{3n+1} = \frac{1}{2} < 1$ C : RATIO TEST

6. $\lim\limits_{n\to\infty} \frac{\sqrt{n}}{n^2+1} \cdot \frac{n^{3/2}}{1} = \lim\limits_{n\to\infty} \frac{n^2}{n^2+1} = 1 \in (0,\infty)$ C : LCT

7. $\lim\limits_{n\to\infty} \frac{100^{n+1}}{(n+1)!} \cdot \frac{(n)!}{100^n} = 100 \lim\limits_{n\to\infty} \frac{1}{n+1} = 0 < 1$ C : RATIO TEST

8. RATIO: $\lim\limits_{n\to\infty} \frac{n^n}{(n+1)^{(n+1)}} = \lim\limits_{n\to\infty} \left(\frac{n}{n+1}\right)^n \cdot \frac{1}{n+1} = \lim\limits_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n} \cdot \frac{1}{n+1} = \lim\limits_{n\to\infty} \left[\cancelto{e}{\left(1+\frac{1}{n}\right)^n}\right]^{-1} \cdot \cancelto{0}{\frac{1}{n+1}}$

$= 0 < 1$ C : RATIO

ROOT: $\lim\limits_{n\to\infty} \sqrt[n]{\frac{1}{n^n}} = \lim\limits_{n\to\infty} \frac{1}{n} = 0 < 1$ C : ROOT

DCT: $\sum_1^\infty \frac{1}{n^n} = 1 + \sum_2^\infty \frac{1}{n^n}$ $\qquad \frac{1}{n^n} \le \frac{1}{n^2}$ on $2, 3, 4, \ldots$

$\therefore \sum_2^\infty \frac{1}{n^n}$ converges

$\therefore 1 + \sum_2^\infty \frac{1}{n^n}$ converges $\quad \therefore \sum_1^\infty \frac{1}{n^n}$ converges.