BC: Q401.CH9A – Convergent and Divergent Series (LESSON 4)

GRAND FINALE REVIEW

Find the interval of convergence of the power series.

1. $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$: Pg. 523 #37

Find the interval of convergence of the power series. *Also state the center and radius of convergence.*

2. $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$: Pg. 523 #43
3. $\sum_{n=0}^{\infty} (-2)^n (n+1)(x-1)^n$: Pg. 523 #47
4. $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{\frac{3}{2}}}$: Pg. 523 #48

Find the interval of convergence of the series and, within this interval, the sum of the series as a function of *x*.

5. $\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}$: Pg. 523 #39
6. $\sum_{n=0}^{\infty} (\ln x)^n$: Pg. 523 #50

Given the power series …

A. Determine the values of *x* for which the power series converges absolutely.
B. Determine the values of *x* for which the power series converges conditionally.
C. Determine the values of *x* for which the power series diverges.

7. $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2n+3}$

**** Positive Term Series Review ****

Determine whether each <u>positive-term-series</u> converges or diverges.
There may be more than one test that yields conclusive results.

1. $\sum_{n=1}^{\infty} \frac{1}{n^4 + n^2 + 1}$

2. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+4}$

3. $\sum_{n=1}^{\infty} \frac{1+2^n}{1+3^n}$

4. $\sum_{n=1}^{\infty} ne^{-n}$

5. $\sum_{n=1}^{\infty} \frac{3n+1}{2^n}$

6. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

7. $\sum_{n=1}^{\infty} \frac{100^n}{n!}$

8. $\sum_{n=1}^{\infty} \frac{1}{n^n}$

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{n}{n+2}$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty} \frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

22. If $\lim_{b\to\infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges

(B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges

(C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges

(E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

76. For what integer k, $k>1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

84. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3<x<-1$ (B) $-3\le x<-1$ (C) $-3\le x\le -1$ (D) $-1\le x<1$ (E) $-1\le x\le 1$

2003 AP EXAM

6) What are all values of p for which $\int_1^\infty \frac{1}{x^{2p}}dx$ converges?

(A) $p < -1$
(B) $p > 0$
(C) $p > \frac{1}{2}$
(D) $p > 1$
(E) There are no values of p for which this integral converges.

10) What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

(A) 1
(B) 2
(C) 4
(D) 6
(E) The series diverges.

22) What are all values of p for which the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^p + 1}$ converges?

(A) $p > 0$
(B) $p \geq 1$
(C) $p > 1$
(D) $p \geq 2$
(E) $p > 2$

24) Which of the following series diverge?

$$I.\ \sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$$

$$II.\ \sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$$

$$III.\ \sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$$

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

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4. Consider the series $\sum_{n=1}^{\infty} \frac{e^n}{n!}$. If the ratio test is applied to the series, which of the following inequalities results, implying that the series converges?

(A) $\lim_{n \to \infty} \frac{e}{n!} < 1$

(B) $\lim_{n \to \infty} \frac{n!}{e} < 1$

(C) $\lim_{n \to \infty} \frac{n+1}{e} < 1$

(D) $\lim_{n \to \infty} \frac{e}{n+1} < 1$

(E) $\lim_{n \to \infty} \frac{e}{(n+1)!} < 1$

12. Which of the following series converges for all real numbers x ?

(A) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(B) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(C) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

(D) $\sum_{n=1}^{\infty} \frac{e^n x^n}{n!}$

(E) $\sum_{n=1}^{\infty} \frac{n! x^n}{e^n}$

16. What are all values of x for which the series $\sum_{n=1}^{\infty} \left(\frac{2}{x^2 + 1} \right)^n$ converges?

(A) $-1 < x < 1$

(B) $x > 1$ only

(C) $x \geq 1$ only

(D) $x < -1$ and $x > 1$ only

(E) $x \leq -1$ and $x \geq 1$

79. Let f be a positive, continuous, decreasing function such that $a_n = f(n)$. If $\sum_{n=1}^{\infty} a_n$ converges to k, which of the following must be true?

(A) $\lim_{n \to \infty} a_n = k$

(B) $\int_1^n f(x)\, dx = k$

(C) $\int_1^{\infty} f(x)\, dx$ diverges.

(D) $\int_1^{\infty} f(x)\, dx$ converges.

(E) $\int_1^{\infty} f(x)\, dx = k$

82. If $\sum_{n=1}^{\infty} a_n$ diverges and $0 \le a_n \le b_n$ for all n, which of the following statements must be true?

(A) $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

(B) $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.

(C) $\sum_{n=1}^{\infty} (-1)^n b_n$ diverges.

(D) $\sum_{n=1}^{\infty} b_n$ converges.

(E) $\sum_{n=1}^{\infty} b_n$ diverges.

2012 AP EXAM

5. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty}\left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

(A) -3 (B) $-\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{13}{16}$ (E) 4

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{8^n}{n!}$ II. $\sum_{n=1}^{\infty}\frac{n!}{n^{100}}$ III. $\sum_{n=1}^{\infty}\frac{n+1}{(n)(n+2)(n+3)}$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

13. What is the radius of convergence of the series $\sum_{n=0}^{\infty}\frac{(x-4)^{2n}}{3^n}$?

(A) $2\sqrt{3}$ (B) 3 (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) 0

22. The power series $\sum_{n=0}^{\infty} a_n(x-3)^n$ converges at $x = 5$. Which of the following must be true?

(A) The series diverges at $x = 0$.

(B) The series diverges at $x = 1$.

(C) The series converges at $x = 1$.

(D) The series converges at $x = 2$.

(E) The series converges at $x = 6$.

27. For what values of p will both series $\sum_{n=1}^{\infty}\frac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty}\left(\frac{p}{2}\right)^n$ converge?

(A) $-2 < p < 2$ only

(B) $-\frac{1}{2} < p < \frac{1}{2}$ only

(C) $\frac{1}{2} < p < 2$ only

(D) $p < \frac{1}{2}$ and $p > 2$

(E) There are no such values of p.

90. If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all n, which of the following must be true?

(A) $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0$

(B) $|a_n| < 1$ for all n

(C) $\sum_{n=1}^{\infty} a_n = 0$

(D) $\sum_{n=1}^{\infty} na_n$ diverges.

(E) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.