BC.Q303.CH8 PRACTICE EXAM

No Calculators

60 Minutes

PART I: L'Hopital's Rule – Find each limit, if it exists. Show all work.

1. $\lim_{x\to 0} \dfrac{e^{2x}-1}{\sin x}$

2. $\lim_{x\to 1^-} (2-x)^{\tan[(\pi/2)x]}$

3. $\lim_{x\to 0} \dfrac{\int_5^{5-x} \dfrac{\sin(t/5)}{2t}\,dt}{x}$

PART II: IMPROPER INTEGRALS – Write an appropriate limit statement(s) to express each integral.
DO NOT EVALUATE

4. $\int_1^4 \dfrac{t-2}{(t+1)(t-4)}\,dt$. *DO NOT EVALUATE*

5. $\int_0^2 \ln x\,dx$ *DO NOT EVALUATE*

6. $\int_{-2}^{1} \dfrac{dx}{x^{2/3}}$ *DO NOT EVALUATE*

Part III: Miscellaneous

7. Let *f* be the function satisfying $f'(x) = -4x^2 f(x)$, for all real numbers *x*, with $f(1) = -2$ and $\lim_{x\to\infty} f(x) = 0$. Evaluate $\int_1^\infty 12x^2 f(x)\,dx$. Show the work that leads to your answer.

PART IV: COMPARISON TESTS – Prove that each of the following converges or diverges. Show all work.

8. $\int_1^\infty \dfrac{1}{2e^x - 3}\,dx$ (USE LCT)

< EXAM ENDS HERE > <SEE NEXT PAGE FOR ADDITIONAL ITEMS>

POST EXAM QUESTION 1 (BEYOND THE 60 MINUTES FOR QUESTIONS 1 – 8)

A. Evaluate: $\int xe^{-x^2}\,dx$

B. Find the value of $\int_0^\infty x^2e^{-x^2}\,dx$, given the fact that $\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$.

Use integration by parts utilizing the antiderivative in part A.

POST EXAM QUESTION 2 (BEYOND THE 60 MINUTES FOR QUESTIONS 1 – 8)

NOW GO AND SEE IF YOU CAN EVALUATE THE INTEGRALS IN NUMBERS 4, 5, and 6 OF THIS EXAM.