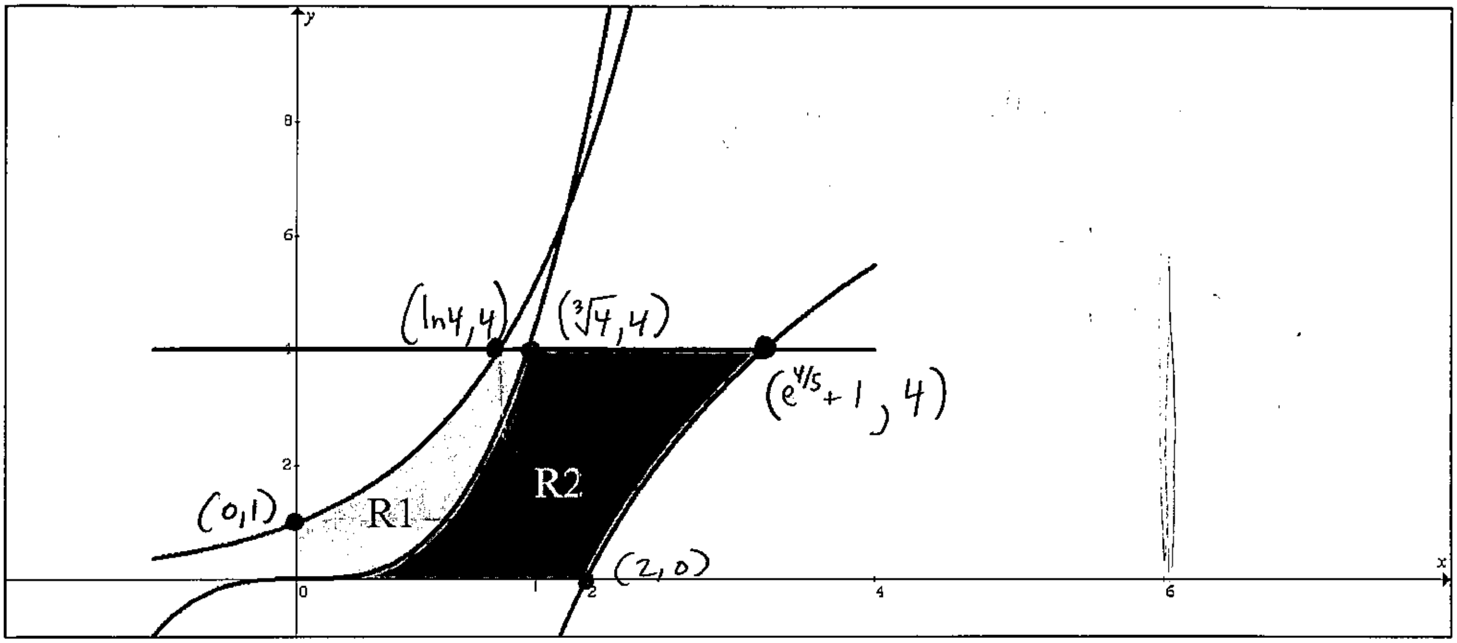
BC: Q302 CH7

Some additional Review:

[Solutions]



1. (NO CALCULATOR) Consider the curves $y = e^x$, $y = x^3$, $y = 5\ln(x-1)$, and $y = 4$ each shown above.

Let R1 be the light shaded region bounded in the first quadrant by the graphs of $y = e^x$, $y = x^3$, and $y = 4$.

Let R2 be the dark shaded region bounded in the first quadrant by the graphs of $y = x^3$, $y = 5\ln(x-1)$, and $y = 4$.

A. Write, but do not evaluate, and expression involving one or more integrals used to find the volume of the solid generated when R2 is rotated about the vertical line $x = 6$.

B. The region R2 is the base of a solid. Every cross section perpendicular to the y-axis is a square whose side lies flat on R2. Write, but do not evaluate, an expression involving one or more integrals that can be used to find the volume of the solid.

C. **FIND** the area of region R2.

D. Write, but do not evaluate, an expression involving one or more integrals, used to find the area of region R1.

E. Write, but do not evaluate, an expression involving one or more integrals, used to find the perimeter of region R2.

Solutions on Next page

1. $y = e^x \rightarrow x = \ln y \qquad y = x^3 \rightarrow x = \sqrt[3]{y} \qquad y = 5\ln(x-1) \rightarrow x = e^{y/5} + 1$

A.

Washers: $V = \pi \int_0^4 \left[(6 - \sqrt[3]{y})^2 - (6 - (e^{y/5} + 1))^2 \right] dy$

(OR)

Cylindrical Shells: $V = \int_0^{\sqrt[3]{4}} 2\pi(6-x)[x^3]\,dx + \int_{\sqrt[3]{4}}^{2} 2\pi(6-x)[4]\,dx + \int_2^{1+e^{4/5}} 2\pi(6-x)[4 - 5\ln(x-1)]\,dx$ (OR)

B. $A_{\square} = (\text{base})^2 = (e^{y/5} + 1 - \sqrt[3]{y})^2$

$V = \int_0^4 A\,dy = \int_0^4 [\underbrace{e^{y/5} + 1 - \sqrt[3]{y}}_{\text{base}}]^2\,dy$

C. $\int_0^4 [e^{y/5} + 1 - \underset{\uparrow\, y^{1/3}}{\sqrt[3]{y}}]\,dy = \left[5e^{y/5} + y - \frac{3}{4}y^{4/3}\right]_0^4 = \left(5e^{4/5} + 4 - \frac{3}{4}(4)^{4/3}\right) - (5)$

D. Δx: $A = \int_0^{\ln 4} [e^x - x^3]\,dx + \int_{\ln 4}^{\sqrt[3]{4}} [4 - x^3]\,dx$

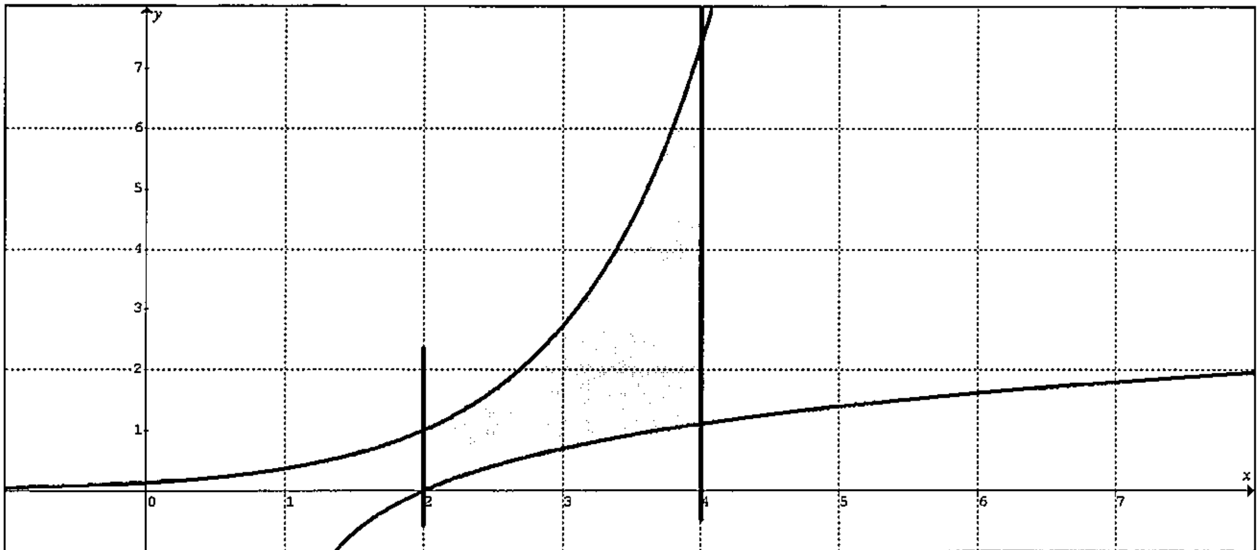
(OR)

Δy: $A = \int_0^1 \sqrt[3]{y}\,dy + \int_1^4 [\sqrt[3]{y} - \ln y]\,dy$

E. Δx: $P = \underbrace{2}_{\text{side 1}} + \underbrace{\int_2^{1+e^{4/5}} \sqrt{1 + \left(\frac{5}{x-1}\right)^2}\,dx}_{\text{side 2}} + \underbrace{\left[(e^{4/5} + 1) - \sqrt[3]{4}\right]}_{\text{side 3}} + \underbrace{\int_0^{\sqrt[3]{4}} \sqrt{1 + (3x^2)^2}\,dx}_{\text{side 4}}$

(or)

Δy: $P = \underbrace{2}_{\text{side 1}} + \underbrace{\int_0^4 \sqrt{1 + \left(\frac{1}{5}e^{y/5}\right)^2}\,dy}_{\text{side 2}} + \underbrace{e^{4/5} + 1 - \sqrt[3]{4}}_{\text{side 3}} + \underbrace{\int_0^4 \sqrt{1 + \left(\frac{1}{3y^{2/3}}\right)^2}\,dy}_{\text{side 4}}$



2 (NO CALCULATOR): Consider the region R bounded by the graphs of $y = e^{(x-2)}$, $y = \ln(x-1)$, $x = 2$, and $x = 4$ as shown by the shaded region above.

$x = 2 + \ln y \qquad x = 1 + e^y$

A. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the x axis.

$$V = \pi \int_2^4 \left[(e^{x-2})^2 - (\ln(x-1))^2 \right] dx \qquad \text{(washer)}$$

B. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the y axis.

$$V = \int_2^4 2\pi (x) \left[e^{x-2} - \ln(x-1) \right] dx \qquad \text{(cyl. shell)}$$

C. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $x = 16$.

$$V = \int_2^4 2\pi (16 - x) \left[e^{x-2} - \ln(x-1) \right] dx \qquad \text{(cyl. shell)}$$

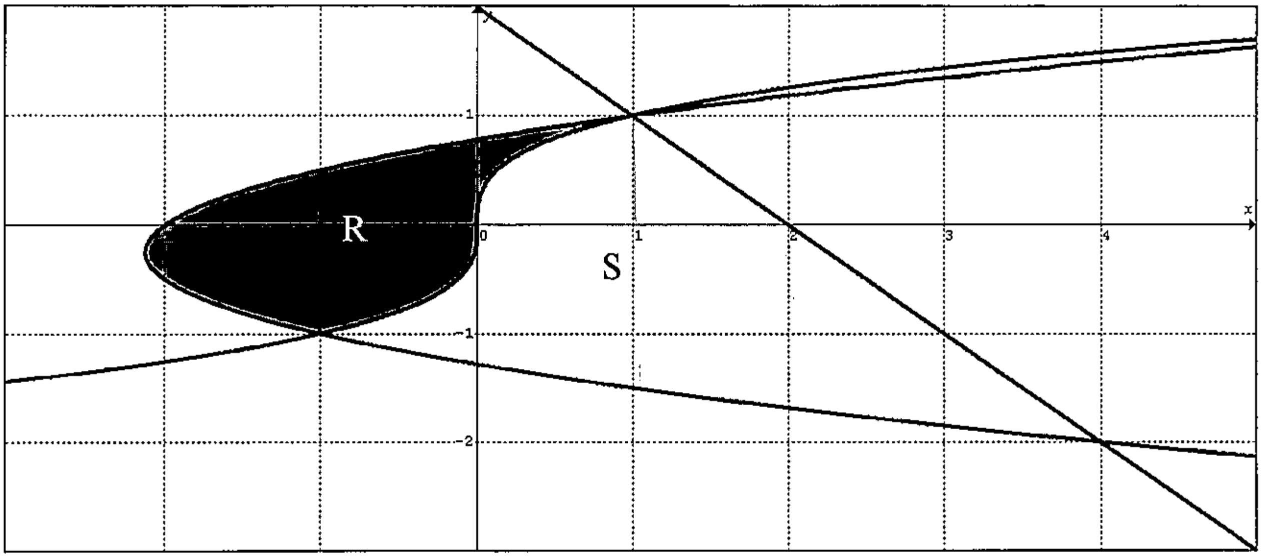
D. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $y = 16$.

$$V = \pi \int_2^4 \left[(16 - \ln(x-1))^2 - (16 - e^{x-2})^2 \right] dx \qquad \text{(washer)}$$

E. Set up, but do not evaluate, an expression involving one or more integrals use to find the perimeter of region R.

$$P = \underbrace{1}_{\text{Side 1}} + \underbrace{\int_2^4 \sqrt{1 + \left(\tfrac{1}{x-1}\right)^2}\, dx}_{\text{Side 2}} + \underbrace{e^2 - \ln(3)}_{\text{Side 3}} + \int_2^4 \sqrt{1 + (e^{x-2})^2}\, dx$$

* $x = 2y^2 + y - 2 \rightarrow$ Cannot solve for $y = f(x)$



$\nearrow x = y^3$

3 (NO CALCULATOR): Consider the region **R** bounded by the graphs of $x = 2y^2 + y - 2$ and $y = \sqrt[3]{x}$ as shown by the dark shaded region above. Consider, also, the region **S** bounded by the graphs of $x = 2y^2 + y - 2$, $y = \sqrt[3]{x}$, and $x + y = 2$ as shown by the light shaded region above.

$x = 2 - y$

A. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $x = 4$.

$$V = \pi\int_{-1}^{1} \left[\left(4 - (2y^2 + y + 2)\right)^2 - \left(4 - y^3\right)^2 \right] dy \quad \text{(Washers)}$$

B. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $y = 4$.

$$V = \int_{-1}^{1} 2\pi (4 - y)\left[y^3 - (2y^2 + y - 2)\right] dy \quad \text{(Cyl. shell)}$$

C. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region S is revolved about the line $x = 4$.

$$V = \pi\int_{-2}^{-1} \left[\left(4 - (2y^2 + y - 2)\right)^2 - \left(4 - (2 - y)\right)^2 \right] dy + \pi\int_{-1}^{1} \left[(4 - y^3)^2 - \left(4 - (2 - y)\right)^2 \right] dy$$

(washer)

D. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region S is revolved about the line $y = 4$.

$$V = \int_{-2}^{-1} 2\pi (4 - y)\left[2 - y - (2y^2 + y - 2)\right] dy + \int_{-1}^{1} 2\pi (4 - y)\left[2 - y - y^3\right] dy$$

(Cyl. shell)