☐ Q301 CH 5 PRACTICE EXAM - CALCULATOR SECTION

1. (a) $\int_0^{12} P(t)\,dt = \Delta t\,[P(2)+P(6)+P(10)] = 4[46+57+62] = 660\ ft^3$

(b) $\int_0^{12} R(t)\,dt = 305\ ft^3$

(c) Amount $= 1000 + 660 - 305 = 1355\ ft^3$

(d) $A(t) = 1000 + \int_0^t [P(u)-R(u)]\,du$

(e) $A'(t) = P(t) - R(t)$ $\quad A'(4) = P(4) - R(4) = 53 - 0.556 \approx 52$

$A'(4)$ is the rate at which the pool water volume is changing at $t=4$ hours.

2. A. Av $W(t) = \frac{1}{15}\int_0^{15} W(t)\,dt \approx \frac{1}{15}[\text{TRAM}] = \frac{1}{15}\left[\frac{3}{2}[20+2(31)+2(28)+2(24)+2(22)+21]\right]$

$= 25.1\ °C$

B. $W'(12)$ is the rate of change in temperature (in °C per day) at time $t = 12$ days.

C. $W'(12) \approx \dfrac{W(15)-W(9)}{15-9} = \dfrac{21-24}{6} = -\frac{1}{2}\ °C/day$

3. FTC-2 (in disguise) $\int_0^3 g'(x)\,dx = g(3) - g(0)$

$2.019 = 7 - g(0)$ $\quad \therefore \boxed{g(0) = 4.981}$

☐ Q301 CH5 PRACTICE EXAM - NON CALCULATOR SECTION

4. $g'(x) = f(x)$ The graph we see is the derivative!

(a) $g(3) = \int_0^3 f(x)\,dx = \frac{\pi(2)^2}{4} - \frac{1}{2}(1) = \boxed{\pi - \frac{1}{2}}$

$*\ g'(x) = f(x) = 0$ at $x = -2, 2, 4$

(b) g has a relative max at $x = 2$ b/c $g'(x) = f(x)$ goes from positive to negative at $x = 2$.

(c) $y - g(3) = g'(3)(x-3) \longrightarrow \boxed{y - (\pi - \tfrac{1}{2}) = -1(x-3)}$

$g'(3) = f(3) = -1$

(d) g has a point of inflection at $x = 0, 3$ b/c $g''(x) = f'(x)$ changes sign at these x values

(e) $x = 4$ is not a candidate $\quad x = -2, 2, 5$ are candidates

endpt $g(-2) = \int_0^{-2} f(x)\,dx = -\int_{-2}^0 f(x)\,dx = -(\pi) = -\pi$ (could have also eliminated $x=-2$ with explanation)

$g(2) = \int_0^2 f(x)\,dx = \pi$

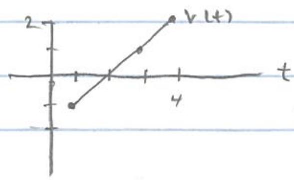
$g(5) = \int_0^5 f(x)\,dx = \pi - \frac{1}{2}(2) + \frac{1}{2} = \pi - \frac{1}{2}$

The absolute max is $\underline{\underline{\pi}}$

(f) $B'(x) = g'(x) + \frac{1}{2} = f(x) + \frac{1}{2} = 0$ when $f(x) = -\frac{1}{2}$ which occurs at $x = 2.5$ and $x = 3.5$. B has a relative minimum at $x = 3.5$ b/c B' goes from negative to positive at $x = 3.5$

5. $x(1) = 5 \quad x'(t) = v(t) = t - 2$

A]



You can appeal to geometry:

$$TD = -\int_1^2 v(t)\,dt + \int_2^4 v(t)\,dt = +\frac{1}{2} + \frac{1}{2}(4) = \frac{5}{2}$$

$$-(-1/2) + (2)$$

B] $$x(4) = x(1) + \int_1^4 x'(t)\,dt = 5 + \underbrace{\int_1^4 v(t)\,dt}_{\text{displacement}} = 5 - \frac{1}{2} + 2 = \frac{13}{2}$$

6. $$\left[e^{(3x^2)^3} - \cos^5(3x^2)\right] 6x$$

chain Rule

7. $u = x^2 - 3$

$x = -1 : u = -2$

$x = 4 : u = 13$

$$\int_{-1}^{4} x(x^2-3)^5\,dx = \frac{1}{2}\int_{-2}^{13} u^5\,du \quad [C]$$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

76 $\quad$ 78 -15 $\overline{63}$

8. $$\int_{-2}^{1}(x^2 - 5x)\,dx = \left[\frac{x^3}{3} - \frac{5x^2}{2}\right]_{-2}^{1} = \left(\frac{1}{3} - \frac{5}{2}\right) - \left(\frac{-8}{3} - \frac{5(4)}{2}\right)$$

$$= \frac{2}{6} - \frac{15}{6} + \frac{16}{6} + \frac{60}{6} = \frac{63}{6} = \boxed{\frac{21}{2}}$$

9. $$\text{AV } f(x) = \frac{\int_{-1}^{2} f(x)\,dx}{2-(-1)} = \frac{1}{3}\int_{-1}^{2} 2x^4\,dx = \frac{2}{3}\left[\frac{x^5}{5}\right]_{-1}^{2} = \frac{2}{3}\left(\frac{32}{5} + \frac{1}{5}\right) = \frac{2}{3}\cdot\frac{33}{5}$$

$$= \frac{22}{5}$$

$f(z) = 2z^4 = \frac{22}{5} \quad z^4 = \frac{11}{5}$ $\quad \boxed{z = \sqrt[4]{\frac{11}{5}}}$ $\quad z = -\sqrt[4]{\frac{11}{5}}$ out of domain