BC: Q301 CH5 PRACTICE EXAMINATION – CALCULATOR SECTION

(For integral approximations, show the integral for which you are approximating)

| $t$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| $P(t)$ | 0 | 46 | 53 | 57 | 60 | 62 | 63 |

1. An above ground swimming pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The above table gives values of $P(t)$ for selected values of $t$. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where
$R(t) = 0.25e^{0.05t^2}$

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate, to the nearest cubic foot, the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.
(b) Calculate, to the nearest cubic foot, the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours. Include the set up when expressing your answer.
(c) Use the results from parts (a) and (b) to approximate the volume of the water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.
(d) Write an equation for $A(t)$, the amount of water in the pool at time $t$.
(e) Find and interpret $A'(4)$, for $A(t)$ defined in part (d).

2.

| $t$ (days) | $W(t)$ (°C) |
|---|---|
| 0 | 20 |
| 3 | 31 |
| 6 | 28 |
| 9 | 24 |
| 12 | 22 |
| 15 | 21 |

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function $W$ of time $t$. The table above shows the water temperature as recorded every 3 days over a 15-day period.

A. Approximate the <u>average temperature</u>, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days. Use correct units when expressing your answer.
B. Using the correct units, explain the meaning of $W'(12)$.
C. Approximate $W'(12)$.

3. Suppose $g(3) = 7$ and $g'(x) = (\sin x)^x$. Estimate $g(0)$ to four decimal places. Show your reasoning.

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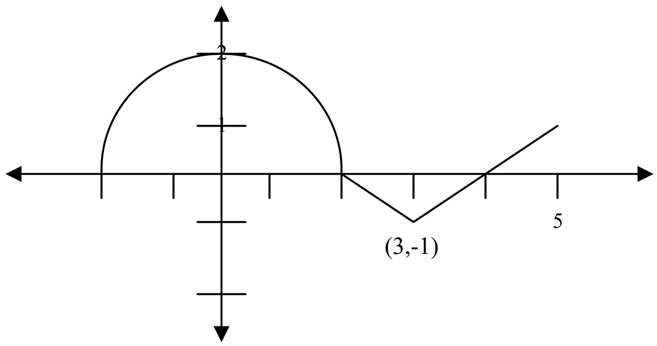


4. The graph of a function *f* consists of a semicircle and two line segments as shown above. Let *g* be the function given by $g(x) = \int_0^x f(t)dt$.

(a) Find $g(3)$.
(b) Find all values of *x* on the open interval (-2, 5) at which *g* has a relative maximum. Justify your answer.
(c) Write an equation for the line tangent to the graph of *g* at $x = 3$.
(d) Find the *x*-coordinate of each point of inflection of the graph of *g* on the open interval (-2, 5). Justify your answer
(e) Find the absolute maximum value of *g* on [-2, 5].
(f) Let $B(x) = g(x) + \frac{1}{2}x$. Find all values of *x* on the open interval (-2, 5) at which *B* has a relative minimum. Justify your answer.

5. A particle starts at position *x* = 5 at time *t* =1 and travels along a horizontal with velocity $x'(t) = t - 2$.
A] What is the total distance traveled by the particle from time *t* = 1 to time *t* = 4? Show work.
B] Where is the particle located at time *t* = 4? Include an integral(s) set up and show work when expressing your answer.

6. Find $\frac{d}{dx}\int_8^{3x^2}\left(e^{t^3} - \cos^5 t\right)dt$.

7.
Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x\left(x^2-3\right)^5 dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5\, du$

(B) $\int_{-2}^{13} u^5\, du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5\, du$

(D) $\int_{-1}^{4} u^5\, du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5\, du$

8. Evaluate $\int_{-2}^{1}\left(x^2 - 5x\right)dx$. Simplify your answer.

9. The Mean Value Theorem for Integrals states: If *f* is continuous on [*a*, *b*], then there is a value $x = z$ on [*a*,*b*], such that $f(z)$ = the average of *f* over [*a*,*b*]. If $f(x) = 2x^4$ on [-1,2], then find the value of *z* that satisfies the conclusion to the mean value theorem.