BC LESSON 4: Q204 – CHAPTER 6: PRACTICE EXAM

#1 (No Calculator) Let $\frac{dy}{dx} = 6(14 - y)$ where $y(0) = 2$.

A. Solve the differential equation by separating the variables

B. Find $\frac{d^2y}{dx^2}$ in terms of *y* and use it to find $\frac{d^2y}{dx^2}$ at $y = 4$.

#2 (No Calculator) Let $\frac{dy}{dx} = 6(14 + y)$ where $y(0) = 2$.

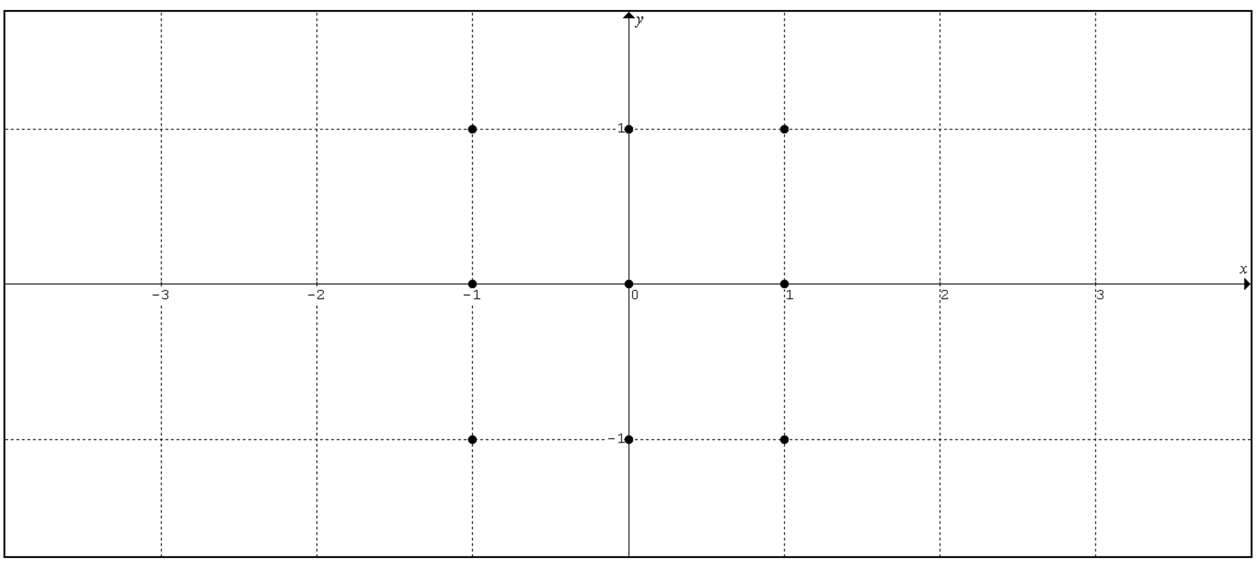
A. Solve the differential equation by separating the variables

B. Find $\frac{d^2y}{dx^2}$ in terms of *y* and use it to find $\frac{d^2y}{dx^2}$ at $y = 4$.

#3 (No Calculator) Let $\frac{dy}{dx} = 3x^2y^2$ where $y(0) = \frac{1}{3}$.

A. Solve the differential equation by separating the variables.

B. Draw a slope for the indicated point in the graph below.

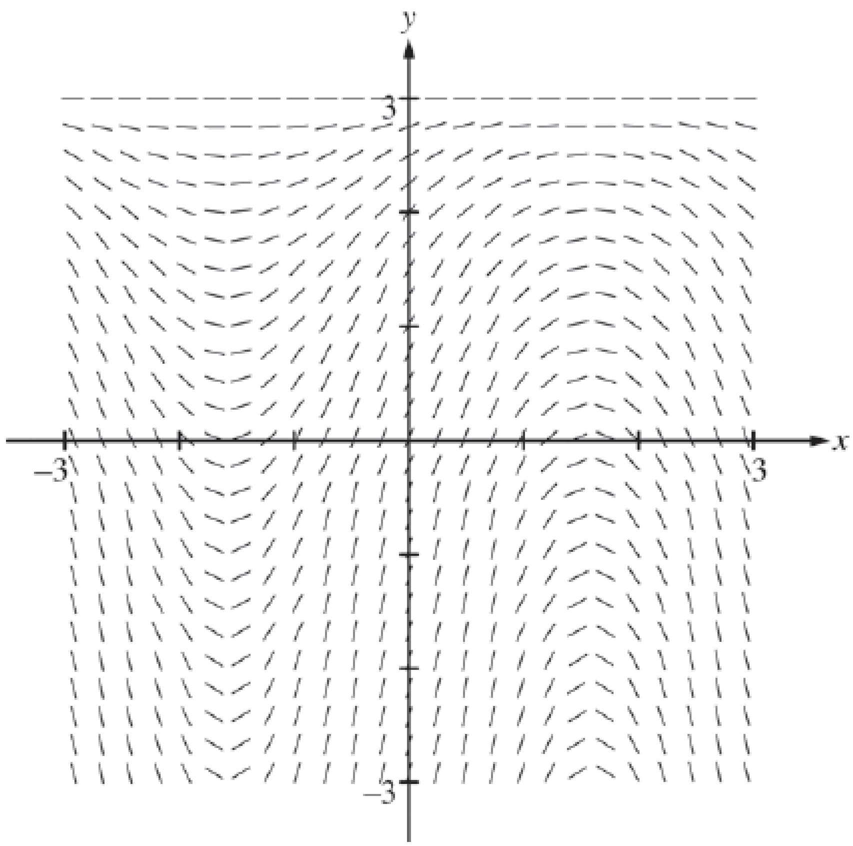


#4 (No Calculator)

Let $\frac{dy}{dx} = x^4 - 3y^2 + 6$ where $y = f(x)$ is the particular solution to the differential equation with the initial condition $f(0) = 1$. Find $f'(0)$ and $f''(0)$.

#5 (No Calculator) Consider the differential equation $\frac{dy}{dx} = (3-y)\cos(x)$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point (0, 1).



(b) Write an equation for the line tangent to the solution curve in part (a) at the point (0, 1). Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = 1$.

#6. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$

Let $y = h(x)$ be the particular solution to the differential equation with $h(0) = 2$.

A. Use a linearization centered at $x = 0$ to approximate $h(1)$.

B. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

C. Is the graph of $y = h(x)$ concave upward or concave downward at $x = 0$? Justify.

#7. Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$

Let $y = f(x)$ be the particular solution to the differential equation with $f(0) = -1$.

A. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

B. Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

#8: If $\frac{dP}{dt} = 0.001P(25 - P)$ and $P(0) = 15$ …

A. Find the function $P(t)$. Provide a quick sketch.

B. Find the value of P when $\frac{dP}{dt}$ is at its greatest.

#9: If $\frac{dP}{dt} = 40P - 5P^2$ and $P(0) = 3$ …

A. Find the function $P(t)$.

B. Find the value of $\frac{dP}{dt}$ when $\frac{dP}{dt}$ is at its greatest.

#10: If $\frac{40}{Z}\frac{dZ}{dt} = 5 - \frac{Z}{60}$ and $Z(0) = 25$ …

A. Find the function $Z(t)$.

B. Find $\lim_{t \to \infty} Z(t)$.

#11. (No Calculator) Let $R(t)$ represent the number of rats in a farm house at time t days. The rate at which the rat population grows is directly proportional to the number of rats present where the constant of proportionality is k.

A. If there were 24 rats initially, find the function $R(t)$ in terms of t and k.
B. If there are 72 rats in 2 days, find k.
C. How many rats would we expect to find after 8 days?

#12. (No Calculator) Let $T(t)$ represent the temperature (degrees Celsius) of tea in a cup at time t minutes, with $t \geq 0$. The temperature $T(t)$ is changing at a rate directly proportional to the difference between the room temperature (20^{o}C) and the temperature of the tea, i.e. $\frac{dT}{dt} = k(20 - T)$ where the constant of proportionality is k.

A. If the tea is poured with an initial temperature of 100 degrees Celsius, find $T(t)$ in terms of t and k.
B. If the tea cools to 60 degrees Celsius in 3 minutes, find k.
C. What will be the temperature of the tea after 6 minutes?
D. At what time will the temperature reach 30 degrees Celsius?