BC – Additional Integration Problems Involving "Integration by Parts"

Evaluate each integral using the method of "integration by parts".

You may wish to use tabular integration by parts if applicable

1. $\int x^2 e^{-x} dx$

2. $\int x^2 \sqrt{x-1} dx$

3. $\int x \sec^2 x dx$

4. $\int \sin^{-1} x dx$

5. $\int \sqrt{x} \ln x dx$

Solutions

AB – Additional Integration Problems Involving "Integration by Parts"

Evaluate each integral using the method of "integration by parts".

You may wish to use tabular integration by parts if applicable

1. $\int x^2 e^{-x}\,dx = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$

x^2 → e^{-x} (+)
$2x$ → $-e^{-x}$ (−)
2 → e^{-x} (+)
0 → $-e^{-x}$

3
15
7
105

2. $\int x^2\sqrt{x-1}\,dx = \frac{2}{3}x^2(x-1)^{3/2} - \frac{8}{15}x(x-1)^{5/2} + \frac{16}{105}(x-1)^{7/2} + C$

x^2 → $\sqrt{x-1} = (x-1)^{1/2}$ (+)
$2x$ → $\frac{2}{3}(x-1)^{3/2}$ (−)
2 → $\frac{4}{15}(x-1)^{5/2}$ (+)
0 → $\frac{8}{15\cdot 7}(x-1)^{7/2}$

3. $\int x\sec^2 x\,dx = x\tan x + \ln|\cos x| + C$

x → $\sec^2 x$ (+)
1 → $\tan x$ (−)
0 → $-\ln|\cos x|$

4. $\int \sin^{-1} x\,dx$

$u = \sin^{-1}x \quad dv = dx$
$du = \frac{1}{\sqrt{1-x^2}} \quad v = x$

$= x\sin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}\,dx$

$u = 1-x^2$
$du = -2x\,dx$
$dx = \frac{du}{-2x}$

$= x\sin^{-1}x + \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$
$= x\sin^{-1}x + \frac{1}{2}u^{1/2}\cdot\frac{2}{1} + C$
$= x\sin^{-1}x + u^{1/2} + C$
$= x\sin^{-1}x + \sqrt{1-x^2} + C$

5. $\int \sqrt{x}\ln x\,dx$

$u = \ln x \quad dv = \sqrt{x}\,dx$
$du = \frac{dx}{x} \quad v = \frac{2}{3}x^{3/2}$

$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}\int x^{1/2}\,dx$
$= \frac{2}{3}x^{3/2}\ln x - \frac{2}{3}x^{3/2}\cdot\frac{2}{3} + C$
$= \frac{2}{3}x^{3/2}\ln x - \frac{4}{9}x^{3/2} + C$