Review: Q202 – BC
Q301 – AB
PRACTICE Solutions

1. $\int \frac{1}{1-x}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C = -\ln|1-x| + C$

$u = 1-x$
$du = -dx$
$dx = -du$

2. $\int \sin^2 x \cos x\,dx = \int u^2\,du = \frac{u^3}{3} + C = \frac{\sin^3 x}{3} + C$

$u = \sin x$
$du = \cos x\,dx$
$dx = \frac{du}{\cos x}$

3. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C$

$u = \sqrt{x}$
$du = \frac{1}{2\sqrt{x}}\,dx$
$dx = 2\sqrt{x}\,du$

4. $\int \frac{1}{1+3x^2}\,dx = \int \frac{1}{1+(\sqrt{3}x)^2}\,dx = \frac{1}{\sqrt{3}}\int \frac{1}{1+u^2}\,du = \frac{1}{\sqrt{3}}\tan^{-1}u + C$
$= \frac{1}{\sqrt{3}}\tan^{-1}(\sqrt{3}x) + C$

$u = \sqrt{3}x$
$du = \sqrt{3}\,dx$
$dx = \frac{du}{\sqrt{3}}$

5. $\int x(1+x^3)\,dx = \int (x + x^4)\,dx = \frac{x^2}{2} + \frac{x^5}{5} + C$ ☺

u – sub does not work!
When in a JAM – EXPAND!

6. $\int x(2-x^2)^3\,dx = -\frac{1}{2}\int u^3\,du = -\frac{1}{2}\frac{u^4}{4} + C = -\frac{1}{8}(2-x^2)^4 + C$

$u = 2-x^2$
$du = -2x\,dx$
$dx = \frac{du}{-2x}$

7. $\int \frac{x^2}{x^3+1}\,dx = \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3+1| + C$

$u = x^3+1$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

8. $\int \frac{x^2}{\sqrt{x^3+1}}\,dx = \frac{1}{3}\int u^{-1/2}\,du = \frac{1}{3}\cdot\frac{2}{1}u^{1/2} = \frac{2}{3}(x^3+1)^{1/2} + C$

$u = x^3+1$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

9. $\int \frac{\sin\left(\frac{3}{x}\right)}{x^2}\,dx = -\frac{1}{3}\int \sin(u)\,du = \frac{+1}{3}\cos(u) + C$

$= \frac{1}{3}\cos\left(\frac{3}{x}\right) + C$

$u = 3/x$

$du = \frac{-3}{x^2}dx$

$dx = \frac{du\,x^2}{-3}$

10. $\int \frac{1}{\sqrt{9-x^2}}\,dx = \int \frac{1}{\sqrt{9}\sqrt{1-\frac{x^2}{9}}}\,dx = \int \frac{1}{3\sqrt{1-\left(\frac{x}{3}\right)^2}}\,dx$

$= \frac{1}{3}\int \frac{dx}{\sqrt{1-\left(\frac{x}{3}\right)^2}} = \frac{1}{3}\cdot 3\int \frac{1}{\sqrt{1-u^2}}\,du$

$u = \frac{x}{3}$

$du = \frac{dx}{3}$

$dx = 3\,du$

$= \sin^{-1}u + C$

$= \sin^{-1}\left(\frac{x}{3}\right) + C$

11. $\frac{d}{dx}\int x^3\sqrt{x-4}\,dx = x^3\sqrt{x-4}$

Cancel out