Solutions

BC: Q201 – EXAMINATION REVIEW (Lessons 1 – 3)

TECHNOLOGY SECTION: *Round answers to three decimal places.*

1. The velocity of a particle moving along a horizontal is given as $v(t) = 8\cos(t) + \ln(\sin(t) + t^2)$ on ~~$0 < t \le 8$.~~ $0.1 < t \le 8$

A. On what time interval is the particle moving to the right? Justify.

you could also use a parenthesis here

$v(t) = 0$ at $t = 1.746$ and $t = 4.343$

moving right on $(\cancel{x}\ 0.1, 1.746) \cup (4.343, 8]$ b/c $v(t) > 0$ on this interval

B. What are the velocity and acceleration at time $t = 5$? Round answers to three decimal places.

$v(5) = 5.449$

$a(5) = 8.099$

C. Is the particle speeding up or slowing down at $t = 3.5$? Justify.

$v(3.5) = -5.015$

$a(3.5) = 3.316$

The particle is slowing down at $t = 3.5$ b/c the velocity and acceleration have different signs at $t = 3.5$

2. The derivative of f is given by $f'(x) = e^{x^2} - 5x^3 + x$ on $0 \le t < 3$

A. On what interval is f decreasing? Justify. $f'(x) = 0$ at $x = 0.824$ and $x = 1.836$

f is decreasing on $[0.824, 1.836]$ b/c $f'(x) < 0$ on $(0.824, 1.836)$

B. At what x-value(s) does f have a relative maximum? Justify. ~~$f'(x) = 0.824$~~ f'(x)=0 at x=0.824

f has a relative max at $x = 0.824$ b/c $f'(x)$ goes from positive to negative at $x = 0.824$

C. On what interval is f concave upward? Justify.

f is concave upward on $(0, 0.344) \cup (1.559, 3)$ b/c $f''(x) > 0$ on this interval.

NO TECHNOLOGY SECTION

1. Let f be defined by $f(x) = \ln(2+\sin x)$ for $\pi \le x \le 2\pi$.
Find the absolute maximum value and the absolute minimum value of f using the closed interval test.

out of domain

$$f'(x) = \frac{1}{2+\sin x} \cdot \cos x = 0 \qquad \cos x = 0 \qquad x = \cancel{\tfrac{\pi}{2}}, \tfrac{3\pi}{2}$$

endpoints at $x = \pi, 2\pi$

$f\left(\frac{3\pi}{2}\right) = \ln(1) = 0$

$f(\pi) = \ln(2)$

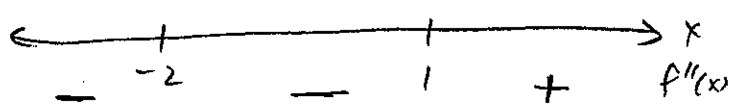
$f(2\pi) = \ln(2)$

The abs. max is $\ln 2$

The abs min is 0

2. A. When is the graph of $f(x)$ concave upward if $f''(x) = (x-1)(x+2)^2 e^{x^2}$. Justify.
B. How many points of inflection are on f? Justify.

$f''(x) = 0 \qquad x = 1, -2$



A. f is concave up on $(1, \infty)$ b/c $f''(x) > 0$ on this interval

B. There is one point of inflection at $x = 1$ b/c $f''(x)$ changes sign at this x-value

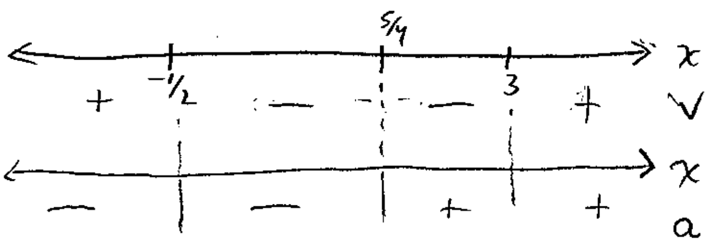
3. A particle moves along a horizontal line. It's position at time t is given as

$$s(t) = \frac{2}{3}t^3 - \frac{5}{2}t^2 - 3t .$$

On what time interval is the particle slowing down? Justify.

$v(t) = 2t^2 - 5t - 3 = 0 \quad (2t+1)(t-3) = 0 \quad t = -\frac{1}{2},\ t = 3$

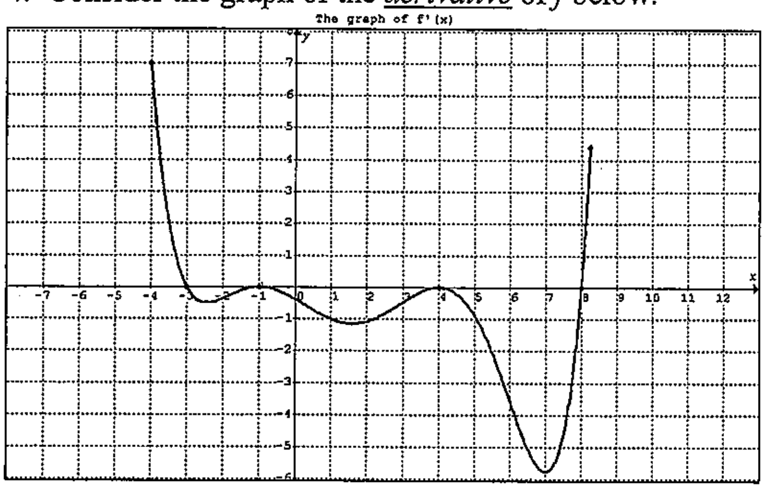
$a(t) = 4t - 5 = 0 \quad t = \frac{5}{4}$



The particle is slowing down on $(-\infty, -\frac{1}{2}) \cup (\frac{5}{4}, 3)$ b/c $v(t)$ and $a(t)$ have opposite signs on this interval

- on $(-\infty, -\frac{1}{2})$ $v(t) > 0$ and $a(t) < 0$
- on $(\frac{5}{4}, 3)$ $v(t) < 0$ and $a(t) > 0$

4. Consider the graph of the *derivative* of f below.



A. For what x – values does f have a local minimum? Justify.

f has a local min at $x = 8$ b/c $f'(x)$ goes from negative to positive at $x = 8$. $f'(x) = 0$ at $x = 8$. Also f has a min at $x = -4$ b/c f is increasing away from the left endpoint

B. On what interval is f increasing? Justify.

f is increasing on $[-4, -3] \cup [8, 8.25]$ (ish) b/c $f'(x) > 0$ on $(-4, -3) \cup (8, 8.25)$

C. On what interval is f concave upward? Justify.

f is concave up on $(-2.5, -1) \cup (1.5, 4) \cup (7, 8.25)$ b/c $f'(x)$ is increasing on this interval.

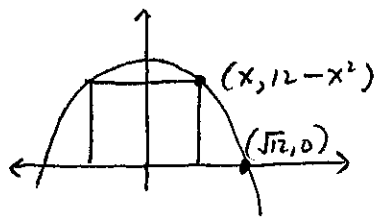
D. How many points of inflection are on f?

f has 5 points of inflection b/c $f''(x)$ changes sign 5 times.

(f' changes its increasing/decreasing behavior 5 times.)

L4–L5 Review Solutions

⑤ Pg. 226 # 6



MAX Area

$A(x) = 2x(12 - x^2)$ $\quad$ $D: \; 0 < x < \sqrt{12}$

$A'(x) = 24 - 6x^2 = 0$

$x = 2$ or $\cancel{x = -2}$ out of domain

$A''(x) = -12x$

$A''(2) = -24 < 0$ $\therefore$ A is max at $x = 2$

$A(2) = 4 \cdot 8 = 32$ units^2

base $= 4$ height $= 8$

⑦ Pg. 229 # 41

$\min D = \sqrt{(x - 3/2)^2 + (\sqrt{x} - 0)^2}$

$Q(x) = D^2 = (x - 3/2)^2 + x$

$Q'(x) = 2(x - 3/2) + 1 = 2x - 2 = 0$

$x = 1$

$Q''(x) = 2$ $\quad$ $Q''(1) = 2 > 0$

$\therefore$ Q is min at $x = 1$.

$\therefore$ D is min at $x = 1$.

$\min D = \sqrt{(1 - 3/2)^2 + (\sqrt{1})^2} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$ units

8) Pg. 258 #52

MAX Area

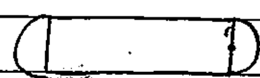
$A(x) = 2x\left(8\cos(0.3x)\right)$ D: $0 < x < 5.236$

$A'(x) = 0$ at $x = 2.868$

A is max at $x = 2.868$ b/c $A'(x)$ goes from positive to negative at this x value.

MAX A $= 29.925\ u^2$

6) C Pg 258. #54



x

$A =$ base (height)

MAX Area rectangle: $A = x(2r)$

$A = (200 - \pi r)(2r)$

$A = 400r - 2\pi r^2$

$A' = 400 - 4\pi r = 0$

$r = \frac{100}{\pi}$

$A'' = -4\pi < 0$

$\therefore$ A is max for $r = \frac{100}{\pi}$

$P = 2x + 2\pi r = 400$

$x + \pi r = 200$

$x = 200 - \pi r$

$r = \frac{100}{\pi}$ $x = 200 - \pi\left(\frac{100}{\pi}\right) = 100$

$r = \frac{100}{\pi}$ units $x = 100$ units

(8) $A(x) = 2x\left(8\cos(0.3x)\right)$ $\quad$ D: $0 < x < 5.236$

$A'(x) = 0$ at $x = 2.868$

A is max at $x = 2.868$ b/c

$A'(x)$ goes from positive to negative at $x = 2.868$

$$\boxed{\text{ANSWER: } A(2.868) = 29.925 \text{ units}^2}$$

(9) Ave rate Δ $[-6, 13/6]$ $= \dfrac{f(13/6) - f(-6)}{\frac{13}{6} - (-6)} = \dfrac{14 - 14}{\frac{13}{6} + 6} = 0$

Inst. rate Δ $= f'(x) = \begin{cases} 2(x+2); & x < 1 \leftarrow [-6, 1) \\ 6; & x \geq 1 \leftarrow [1, 13/6] \end{cases}$

$f'(x) = 0$ $\quad 2x + 4 = 0$ $\quad \boxed{x = -2} \in (-6, 1) \in (-6, 13/6)$

or

$\cancel{6 = 0}$

(10) Ave rate Δ $[-2, 1]$ $= \dfrac{f(1) - f(-2)}{1 - (-2)} = \dfrac{2.334}{3} = 0.778$

Inst rate Δ $= f'(x) = 0.785$ at $\underbrace{x = -1.310 \text{ or } x = 0.344}_{\text{which both belong to the interval } (-2, 1) \checkmark}$

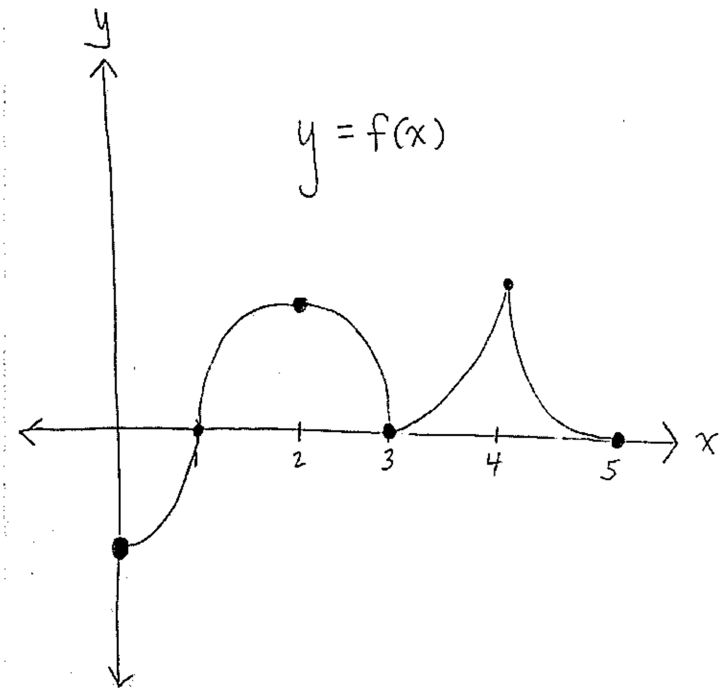
$y1 = \tan^{-1}(e^{x+2}) + \sin(x^2)$

$\left.\begin{array}{l} y2 = \text{<paste derivative of } y1(x)\text{>} \\ y3 = 0.785 \end{array}\right\}$ Find points of intersection

(OR) $y2 = \text{<paste derivative of } y1(x)\text{>} - 0.778 \}$ Find zeros

GRAPH THEORY

11. Below is Steven's graph of $y = f(x)$.



THE CHART REPRESENTS STEVEN'S GRAPH

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$	4	$4<x<5$	5
$f(x)$	−	−	0	+	+	+	0	+	+	+	0
$f'(x)$	0	+	DNE	+	0	−	DNE	+	DNE	−	0
$f''(x)$		+	DNE	−	−	−	DNE	+	DNE	+	

FILL IN EACH BLANK IN THE CHART ABOVE WITH ONE OF THE FOLLOWING:

+ for positive

– for negative

0 for zero

DNE for Does not Exist