3.8 3.9 Review Solutions

① $\frac{dy}{dx} = e^{\cos^{-1}(\tan(2x))} \cdot \frac{-1}{\sqrt{1-(\tan(2x))^2}} \cdot \sec^2(2x) \cdot 2 + 1$

② $\frac{dy}{dx} = 7^{\cos^{-1}\left(\frac{1}{x}\right)} \cdot \ln(7) \cdot \frac{-1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot \frac{-1}{x^2}$

This is $\left(\frac{1}{x}\right)'$

$= \ln(7) \cdot 7^{\cos^{-1}\left(\frac{1}{x}\right)} \cdot \frac{-1}{\sqrt{\frac{x^2-1}{x^2}}} \cdot \frac{-1}{x^2}$

$= \ln(7) \cdot 7^{\cos^{-1}\left(\frac{1}{x}\right)} \cdot \frac{1}{\frac{\sqrt{x^2-1}}{x}} \cdot \frac{1}{x^2}$

$= \boxed{\frac{\ln(7) \cdot 7^{\cos^{-1}\left(\frac{1}{x}\right)}}{x\sqrt{x^2-1}}}$

③ $\frac{dy}{dx} = \frac{1}{2x^4+7x} \cdot (8x^3+7)$

④ Product Rule : $\frac{dy}{dx} = e^{3x} \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2}x^{-1/2} + \tan\sqrt{x} \cdot e^{3x} \cdot 3$

⑤ $\frac{dy}{dx} = \frac{-1}{1+(\sqrt{x})^2} \cdot \frac{1}{2}x^{-1/2} = \frac{-1}{1+x} \cdot \frac{1}{2\sqrt{x}}$

$= \boxed{\frac{-1}{2\sqrt{x}(x+1)}}$

⑥ $\frac{dy}{dx} = \frac{1}{|\tan(2x+1)|\sqrt{\tan^2(2x+1)-1}} \cdot \sec^2(2x+1) \cdot 2$

⑦ $\frac{dy}{dx} = e^{\tan^{-1}(\sin^{-1}x)} \cdot \frac{1}{1+(\sin^{-1}x)^2} \cdot \frac{1}{\sqrt{1-x^2}}$

(8) Product Rule:

$$\frac{dy}{dx} = e^{\tan^{-1}(5x)} \cdot \frac{1}{\csc^{-1}(2x)} \cdot \frac{-1}{|2x|\sqrt{(2x)^2-1}} \cdot 2 + \ln\left(\csc^{-1}(2x)\right) \cdot e^{\tan^{-1}(5x)} \cdot \frac{1}{1+(5x)^2} \cdot 5$$

(9) Quotient Rule:

$$\frac{dy}{dx} = \frac{\sin^{-1}\left(\frac{1}{x^2}\right)\cdot\left(-\sin(e^x)\right)\cdot e^x - \cos(e^x)\frac{1}{\sqrt{1-\left(\frac{1}{x^2}\right)^2}} \cdot \frac{-2}{x^3}}{\left(\sin^{-1}\left(\frac{1}{x^2}\right)\right)^2}$$

(10) $$\frac{dy}{dx} = \frac{1}{\ln(5x)+x}\left[\frac{1}{5x}\cdot 5 + 1\right]$$

(11) $$f(x) = \frac{1}{5}\ln\left(\frac{(x-3)^4(x^2+1)}{(2x+5)^3}\right)$$

$$= \frac{1}{5}\left[\ln(x-3)^4 + \ln(x^2+1) - \ln(2x+5)^3\right]$$

$$= \frac{1}{5}\left[4\ln(x-3) + \ln(x^2+1) - 3\ln(2x+5)\right]$$

$$f'(x) = \frac{4}{5}\cdot\frac{1}{x-3} + \frac{1}{5}\cdot\frac{1}{x^2+1}\cdot 2x - \frac{3}{5}\cdot\frac{1}{2x+5}\cdot 2$$

(12) $$f(x) = \frac{\ln(\sin x + 5)}{\ln(7)} \qquad f'(x) = \frac{1}{\ln(7)}\cdot\frac{1}{\sin x + 5}\cdot\cos x$$

(13) $$f(x) = \frac{\ln(\sec^2 x)}{\ln(10)} = \frac{2\ln(\sec x)}{\ln(10)}$$

$$f'(x) = \frac{2}{\ln(10)}\cdot\frac{1}{\sec x}\cdot\sec x\tan x = \boxed{\frac{2\tan x}{\ln(10)}}$$

(14) $$\ln y = \cos^{-1}x \cdot \ln(x)$$

$$\frac{1}{y}\frac{dy}{dx} = \cos^{-1}x \cdot \frac{1}{x} + \ln x\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = y\left(\frac{\cos^{-1}x}{x} - \frac{\ln x}{\sqrt{1-x^2}}\right)$$

$$\frac{dy}{dx} = x^{\cos^{-1}x}\left(\frac{\cos^{-1}x}{x} - \frac{\ln x}{\sqrt{1-x^2}}\right)$$

(15) $$\ln y = \sin^{-1}x \cdot \ln(\sin x)$$

$$\frac{1}{y}\frac{dy}{dx} = \sin^{-1}x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = (\sin x)^{(\sin^{-1}x)}\left[\frac{\sin^{-1}x \cos x}{\sin x} + \frac{\ln(\sin x)}{\sqrt{1-x^2}}\right]$$

(16) $f'(x) = 21x^6 + 2$

$f'(x) > 0 \quad \forall x$

$\therefore$ f is always increasing

$\therefore$ f is one to one

$\therefore$ The inverse of f is also a function

$$g'(10) = \frac{1}{f'(1)} = \frac{1}{21(1)^6+2} = \boxed{\frac{1}{23}}$$

$3a^7 + 2a + 5 = 10$

$3a^7 + 2a - 5 = 0$

$\therefore a = 1$

(17) $f'(x) = 2 + \cos x$

$2a + \sin(a) = \pi + 1$

$a = \frac{\pi}{2}$

$$\frac{d}{dx}\left[f^{-1}(\pi+1)\right] = \frac{1}{f'(\frac{\pi}{2})} = \frac{1}{2+\cos(\frac{\pi}{2})} = \boxed{\frac{1}{2}}$$

(18) $x\frac{dy}{dx} + y + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 2y\frac{dy}{dx}$

$$\frac{dy}{dx}\left(x + \frac{1}{\sqrt{1-y^2}} - 2y\right) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x + \frac{1}{\sqrt{1-y^2}} - 2y} = \frac{-y}{\frac{x\sqrt{1-y^2}}{\sqrt{1-y^2}} + \frac{1}{\sqrt{1-y^2}} - \frac{2y\sqrt{1-y^2}}{\sqrt{1-y^2}}}$$

$$= \frac{-y}{\frac{x\sqrt{1-y^2} + 1 - 2y\sqrt{1-y^2}}{\sqrt{1-y^2}}} = \frac{-y\sqrt{1-y^2}}{x\sqrt{1-y^2} + 1 - 2y\sqrt{1-y^2}} = \boxed{\frac{y\sqrt{1-y^2}}{2y\sqrt{1-y^2} - x\sqrt{1-y^2} - 1}}$$

(19) $e^{xy}\left(x\frac{dy}{dx} + y\right) - \sin y\frac{dy}{dx} = \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$

$$xe^{xy}\frac{dy}{dx} + ye^{xy} - \sin y\frac{dy}{dx} = \frac{1}{x+y} + \frac{1}{x+y}\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(xe^{xy} - \sin y - \frac{1}{x+y}\right) = \frac{1}{x+y} - ye^{xy}$$

$$\frac{dy}{dx} = \frac{\frac{1}{x+y} - ye^{xy}}{xe^{xy} - \sin y - \frac{1}{x+y}}$$

(20) $x=1$: $\quad e^y + y + 3 = 4 \qquad e^y + y = 1 \quad \because e^0 = 1$

$\therefore y = 0$

$$e^y\frac{dy}{dx} + \frac{dy}{dx} + 3 = 0 \qquad \frac{dy}{dx} = \frac{-3}{e^y+1}$$

$$\frac{dy}{dx}(e^y+1) = -3 \qquad \left.\frac{dy}{dx}\right|_{y=0} = \frac{-3}{1+1} = \frac{-3}{2}$$

$$\boxed{y = \frac{-3}{2}(x-1)}$$