BC: Q103

~~Q201~~ EXAM REVIEW SOLUTIONS

3.7 #1 $y^2 = x^2 - x - 8$

A] $2y\frac{dy}{dx} = 2x - 1 \qquad \frac{dy}{dx} = \frac{2x-1}{2y}$

B] $y = 2 \qquad 4 = x^2 - x - 8 \longrightarrow x^2 - x - 12 = 0 \rightarrow (x-4)(x+3) = 0$

$(4,2) \quad (-3,2) \qquad\qquad x = 4, -3$

C] $\left.\frac{dy}{dx}\right|_{(4,2)} = \frac{8-1}{4} = \frac{7}{4} \longrightarrow y - 2 = 7/4\,(x-4)$

$\left.\frac{dy}{dx}\right|_{(-3,2)} = \frac{-6-1}{4} = \frac{-7}{4} \longrightarrow y - 2 = -7/4\,(x+3)$

D] HORIZONTAL TANGENT $\Rightarrow \frac{dy}{dx} = 0 \qquad \frac{2x-1}{2y} = 0$ when $2x - 1 = 0 \rightarrow x = 1/2$

<BUT> $y^2 = (\frac{1}{2})^2 - (\frac{1}{2}) - 8 \rightarrow y^2 = \frac{-33}{4}$ <BUT> $y^2 \neq \frac{-33}{4}$

E] $\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{2x-1}{2y}\right] = \frac{2y(2) - (2x-1)2\boxed{\frac{dy}{dx}}}{(2y)^2} = \frac{4y - (2x-1)2\left[\frac{2x-1}{2y}\right]}{(2y)^2}$ (SUB)

F] $\left.\frac{d^2y}{dx^2}\right|_{(-3,-2)} = \frac{-8 - (-6-1)(2)\left[\frac{-6-1}{-4}\right]}{(-4)^2} = \frac{-8 - (-7)(2)(\frac{-7}{-4})}{16} = \frac{-8 + \frac{49}{2}}{16}$

$= \frac{-16+49}{32} = \boxed{\frac{33}{32}}$

#2 $x^3 - xy^3 = 18xy$

$3x^2 - x(3y^2)\frac{dy}{dx} + y^3(-1) = 18x\frac{dy}{dx} + y(18)$

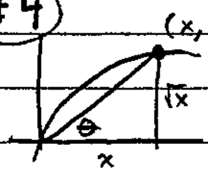
$(-3xy^2 - 18x)\frac{dy}{dx} = 18y + y^3 - 3x^2 \longrightarrow \boxed{\frac{dy}{dx} = \frac{18y + y^3 - 3x^2}{-3xy^2 - 18x}}$

or

$\frac{3x^2 - 18y - y^3}{18x + 3xy^2}$

4.6

#4



Given $\frac{dx}{dt} = 8$ m/s

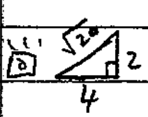
Find $\frac{d\theta}{dt}$ when $x = 4$

Relationship $\tan\theta = \frac{\sqrt{x}}{x}$ — simplifies → $\tan\theta = x^{-1/2}$

••• HARDER WAY •••

$\sec^2\theta\frac{d\theta}{dt} = \frac{x\cdot\frac{1}{2}x^{-1/2}\frac{dx}{dt} - \sqrt{x}\frac{dx}{dt}}{x^2}$

$\sec^2\theta\frac{d\theta}{dt} = -1/2\, x^{-3/2}\frac{dx}{dt}$



$\left(\frac{\sqrt{20}}{4}\right)^2\frac{d\theta}{dt} = -\frac{1}{2}(4)^{-3/2}(8)$

$\cos\theta = \frac{4}{\sqrt{20}} \qquad \frac{20}{16}\frac{d\theta}{dt} = -\frac{1}{2}\left(\frac{1}{8}\right)(8)$

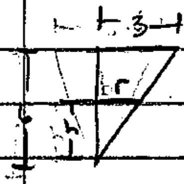
$\sec\theta = \frac{\sqrt{20}}{4} \qquad d\theta/dt = -\frac{1}{2}\cdot\frac{16}{20} = \frac{-8}{20} = \frac{-2}{5}$ radians/sec

(5.) A] CONE: Given| $\frac{dV}{dt} = -10 \text{ in}^3/\text{min}$

Find| dh/dt when $h = 5$

Rel| $V = \frac{1}{3}\pi r^2 h$

UPDATE| $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$



$* \; V = \frac{\pi}{12} h^3$

$\frac{3}{6} = \frac{r}{h}$ $\qquad \frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$6r = 3h$ $\qquad -10 = \frac{\pi}{4}(5)^2 \frac{dh}{dt}$

$r = \frac{h}{2}$ $\qquad \frac{dh}{dt} = \frac{-40}{25\pi} = \frac{-8}{5\pi}$ in/min ← h is decreasing

B] CAN: Given| $\frac{dV}{dt} = +10 \text{ in}^3/\text{min}$

Find| $\frac{dh}{dt}$ when h = DOES NOT MATTER

Rel| $V = \pi r^2 h$ BUT "r" always $= 3$

$* \; V = 9\pi h$

$\frac{dV}{dt} = 9\pi \frac{dh}{dt}$ ... see does not depend on h

$10 = 9\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{10}{9\pi}$ in/min ← h is increasing

#3

$$2\cos(xy^2) + y = x^2 y$$

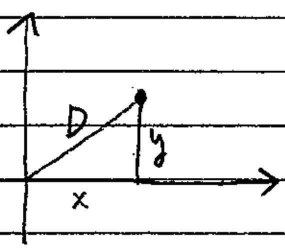
$$-2\sin(xy^2)\left[x \cdot 2y\frac{dy}{dx} + y^2(1)\right] + \frac{dy}{dx} = x^2\frac{dy}{dx} + y(2x)$$

$$-4xy\sin(xy^2)\frac{dy}{dx} - 2y^2\sin(xy^2) + \frac{dy}{dx} = x^2\frac{dy}{dx} + 2xy$$

$$\frac{dy}{dx}\left(-4xy\sin(xy^2) + 1 - x^2\right) = 2xy + 2y^2\sin(xy^2)$$

$$\frac{dy}{dx} = \frac{2xy + 2y^2\sin(xy^2)}{-4xy\sin(xy^2) + 1 - x^2}$$

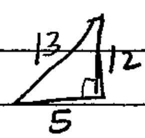
#6.



Given: $\frac{dx}{dt} = -1$ m/sec $\quad \frac{dy}{dt} = 5$ m/sec

Find: $\frac{dD}{dt}$ when $x = 5,\ y = 12$

Rel: $x^2 + y^2 = D^2$



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2D\frac{dD}{dt}$$

$$\cancel{2}(5)(-1) + \cancel{2}(12)(5) = \cancel{2}(13)\frac{dD}{dt}$$

$$\frac{dD}{dt} = \frac{-5 + 60}{13} = \frac{55}{13} \text{ m/s}$$

Section 4.5:

7. Let f be a function with $f(4) = 1.8$ and $f'(x) = \sqrt{5+x}$.
Use a linearization of f at $x = 4$ and use it to approximate $f(4.2)$.

8. Estimate the change in $f(x) = x^3 + 2x$ as x decreases from 3 to 2.8

9. Estimate $\sqrt{8.9}$ using a linearization.

7. $L(x) = f(4) + f'(4)(x-4)$

$L(x) = 1.8 + 3(x-4)$

$f(4.2) \approx L(4.2) = 1.8 + 3(0.2) = \boxed{2.4}$

8. $a = 3 \quad dx = -0.2 \quad f'(x) = 3x^2 + 2$

$\Delta f \approx df = f'(3)\,dx = \left(3(3)^2 + 2\right)(-0.2)$

$= (29)(-0.2)$

$= \boxed{-5.8}$

9. Let $f(x) = \sqrt{x}$

$a = 9 \quad f'(x) = \frac{1}{2\sqrt{x}}$

$L(x) = f(9) + f'(9)(x-9)$

$L(x) = 3 + \frac{1}{6}(x-9)$

$\sqrt{8.9} = f(8.9) \approx L(8.9) = 3 + \frac{1}{6}(-0.1)$

$= 3 - \frac{1}{60}$

$= \boxed{2\frac{59}{60}}$ or $\boxed{\frac{179}{60}}$