PRACTICE EXAM

BC.Q101.EXAMINATION – FORM A

Ch 2.4, 3.1, 3.2: Derivative Foundation

NO CALCULATORS

[60 minutes]

NAME: Solutions

DATE:

BLOCK:

1[10]. Consider the function $k(x)=\begin{cases}2x+4; x\le 1\\ x^2-4x+9; x>1\end{cases}$.

Formally prove that k is or is not **continuous at $x = 1$.**

i) $k(1) = 2(1)+4 = 6$

ii) $\lim_{x\to 1^+} k(x) = \lim_{x\to 1^+} x^2-4x+9 = 6$

$\lim_{x\to 1^-} k(x) = \lim_{x\to 1^-} 2x+4 = 6$

$\therefore \lim_{x\to 1} k(x) = 6$

iii) $\lim_{x\to 1} k(x) = k(1)$

$\therefore$ k is continuous at $x=1$

2[15]. Suppose $f(x)=\begin{cases}2x-3; x\ge 1\\ x^2-2; x<1\end{cases}$.

Formally prove that $f(x)$ is or is not **differentiable at** $x=1$.

Does $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ exist?

□ $f'_+(1) = \lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{2(1+h)-3-[-1]}{h}$

$= \lim_{h\to 0^+} \frac{2h}{h} = \lim_{h\to 0^+} 2 = 2$

□ $f'_-(1) = \lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{(1+h)^2-2-[-1]}{h}$

$= \lim_{h\to 0^-} \frac{1+2h+h^2-2+1}{h} = \lim_{h\to 0^-} 2+h = 2$

$\therefore \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = 2$ (i.e, it exists)

so f is differentiable at $x=1$

3[5]. Consider the continuous and differentiable function $f(x)=\begin{cases}2x+4;\ x\geq 1\\ x^2+5;\ x<1\end{cases}$.

Find the **average rate of change** of f on [-2,3]. Show work.

$$\text{Ave rate}\Delta\ [-2,3]=\frac{f(3)-f(-2)}{3-(-2)}=\frac{[2(3)+4]-[(-2)^2+5]}{5}=\frac{1}{5}$$

4[20]. Let $g(x)$ be a smooth and continuous function that is not explicitly defined, but whose select values are shown in the table below. The domain for $g(x)$ is $[-4,6]$.

x	-4	-3	-2	0	3	4	5	6
$g(x)$	2	5	0	-2	4	6	-12	-15
$g'(x)$	?	?	?	?	1.8	?	?	?

A. **Estimate** $g'(-3)$, $g'(4.5)$. Show work.

$$g'(-3)\approx\frac{g(-2)-g(-4)}{-2-(-4)}=\frac{0-2}{2}=-1$$

$$g'(4.5)\approx\frac{g(5)-g(4)}{5-4}=\frac{-12-6}{1}=-18$$

average on a small neighborhood

B. Write an equation of the line tangent to $g(x)$ at $x=3$.

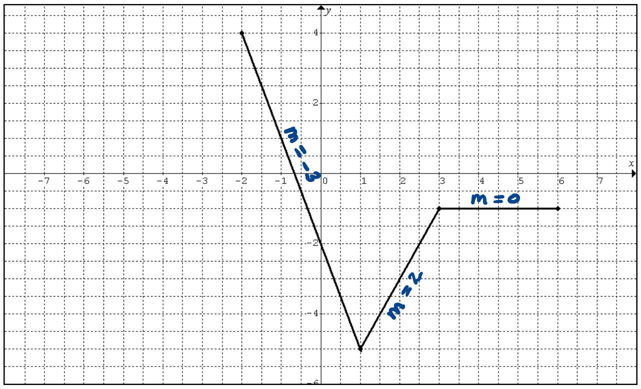
$g'(3)=1.8$ Given $g(3)=4$ given

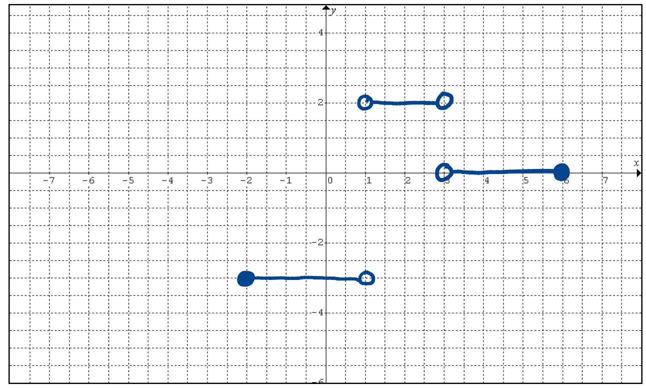
$$y-4=1.8(x-3)$$

C. Find the average rate of change in g on $[-4,6]$. Show work.

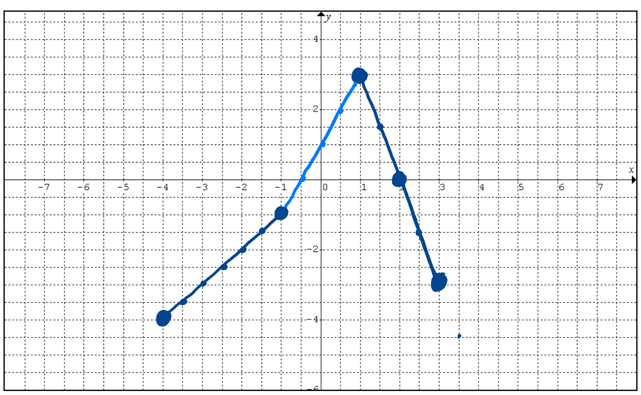
$$\text{Ave rate}\Delta\ [-4,6]=\frac{g(6)-g(-4)}{6-(-4)}=\frac{-15-2}{10}=\frac{-17}{10}$$

5[10]. The graph of $f(x)$ is given below on the left. **Draw** the function $f^{/}(x)$.

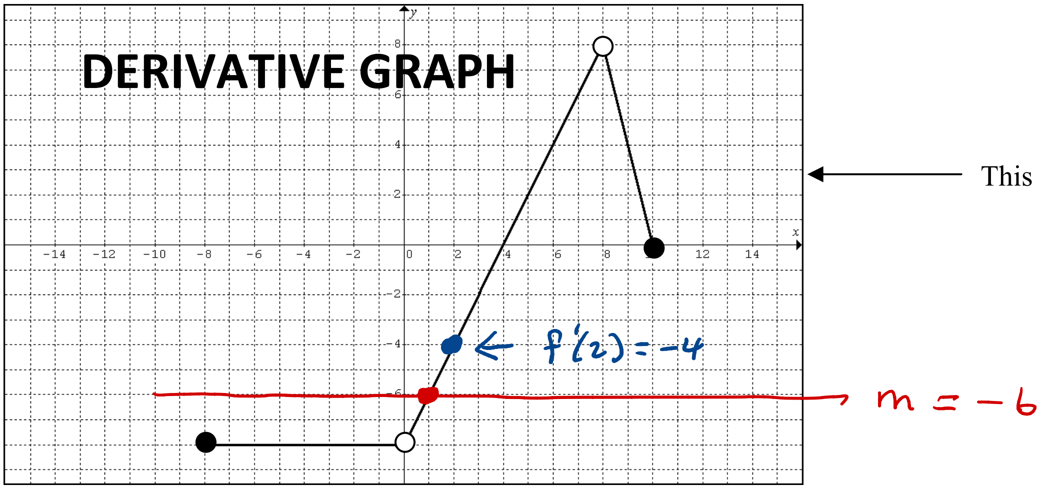




6[10]. **Draw** the function $g(x)$ which is continuous for all points on its domain. The domain of $g(x)$ is [-4, 3], $g(2)=0$ and $g^{/}(x)=\begin{cases} 1; x<-1 \\ 2; -1<x<1 \\ -3; x>1 \end{cases}$.



7[10]. The graph of the ***derivative* of** $f(x)$ is given below.



This is the graph of $f'(x)$

A. If $f(2) = 3$, write an equation of the tangent to the f at $x = 2$

$f'(2) = -4$ ← This is a point on $f'(x)$ graph

$\boxed{y - 3 = -4(x-2)}$

B. For what value(s) of x will f have a horizontal tangent?

where does $f'(x) = 0$?

graph of $f'(x) = 0$ at $x = 4$, $x = 10$

C. For what value(s) of x will f have a tangent line parallel to $y = -6x - 15$ → $m = -6$

where does $f'(x) = -6$

This happens at $x = 1$

8[15]. Let $f(x)=\dfrac{1}{x+1}$.

A. Use the **definition for the derivative at** $x=a$ to find $f'(2)$.

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{\frac{1}{2+h+1}-\frac{1}{3}}{h}=\lim_{h\to 0}\frac{\frac{3}{3(3+h)}-\frac{(3+h)}{3(3+h)}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{-h}{3(3+h)}}{h}=\lim_{h\to 0}\frac{-1}{3(3+h)}=\frac{-1}{9}$$

[OR]

$$f'(2)=\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=\lim_{x\to 2}\frac{\frac{1}{x+1}-\frac{1}{3}}{x-2}$$

$$=\lim_{x\to 2}\frac{\frac{3}{3(x+1)}-\frac{x+1}{3(x+1)}}{x-2}=\lim_{x\to 2}\frac{\frac{2-x}{3(x+1)}}{x-2}$$

$$=\lim_{x\to 2}\frac{-1}{3(x+1)}=\frac{-1}{9}$$

B. Write an **equation** for the line **tangent** to $f(x)$ at $x=2$.

$f(2)=\frac{1}{3}$ $\quad f'(2)=-1/9$

$$y-\frac{1}{3}=-\frac{1}{9}(x-2)$$

9[5]. Suppose that f has the property $f(x+y)=f(x)f(y)$ for all values of x and y and that $f(0)=f'(0)=1$. Show that f is differentiable and $f'(x)=f(x)$.

To show f is differentiable, we must show that $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ exists for all x.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{f(x)f(h)-f(x)}{h}$$

$$= \lim_{h\to 0} f(x) \cdot \underbrace{\lim_{h\to 0} \frac{f(h)-1}{h}}_{\text{we know this exists}}$$

b/c

$$f'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{f(h)-1}{h}$$

$$= 1 \quad \text{(Given)}$$

$$= f(x) \cdot f'(0) \quad \Big\} \text{ exists } \therefore \text{ differentiable}$$

$$= f(x) \cdot 1$$

$$= f(x) \qquad \text{Q.E.D.}$$