5. $\vec{a} = \langle 1, -1, -1 \rangle \quad \vec{b} = \langle \frac{1}{2}, 1, \frac{1}{2} \rangle$

$$\begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ \frac{1}{2} & 1 & \frac{1}{2} \end{vmatrix} = i\left(-\tfrac{1}{2}+1\right) - j\left(\tfrac{1}{2}+\tfrac{1}{2}\right) + k\left(1+\tfrac{1}{2}\right)$$

$$= \tfrac{1}{2}\hat{i} - \hat{j} + \tfrac{3}{2}\hat{k} = \langle \tfrac{1}{2}, -1, \tfrac{3}{2} \rangle = \vec{v}$$

$\vec{a} \cdot \vec{v} = 0 \quad \vec{b} \cdot \vec{v} = 0$

13. (a) yes (b) NO
(c) yes (d) NO
(e) NO (f) yes

9. $(\hat{i} \times \hat{j}) \times \hat{k} = \hat{k} \times \hat{k} = 0$

10. $\hat{k} \times (\hat{i} - 2j) = \hat{k} \times (\hat{i} + -2j)$ parallel

$= (\hat{k} \times \hat{i}) + (\hat{k} \times -2j)$

$= j + -2(\hat{k} \times \hat{j})$

$= j + -2(-\hat{i})$

$= 2i + j$

14. $|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin 60^\circ = (5)(10) \sin 60^\circ$

$= (5)(10)\frac{\sqrt{3}}{2} = 25\sqrt{3}$

⟨INTO PAGE⟩

15. $|\vec{w} \times \vec{v}| = (6)(8)\sin(30^\circ) = (6)(8)\frac{1}{2} = 24$

TRICKY! INTO PAGE "make sure to make tail to tail!

16. a) $|\vec{a} \times \vec{b}| = (3)(2) \sin 90^\circ = 6$

b) $\vec{a} = \langle +, +, 0 \rangle \quad \vec{b} = \langle 0, 0, + \rangle$

$$\begin{matrix} i & j & k \\ + & + & 0 \\ 0 & 0 & + \end{matrix} \qquad \langle +, -, 0 \rangle$$

17. $$\begin{matrix} i & j & k \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{matrix} \qquad i(6-1) - j(3-0) + k(1-0)$$

$= 5i - 3j + k = \langle 5, -3, 1 \rangle = a \times b$

$b \times a = -\langle 5, -3, 1 \rangle = \langle -5, 3, -1 \rangle$

19. $\langle 1,-1,1\rangle \times \langle 0,4,4\rangle = \begin{matrix} i & j & k \\ 1 & -1 & 1 \\ 0 & 4 & 4 \end{matrix}$

$(-4-4)i - (4-0)j + (4-0)\hat{k}$

$\begin{matrix} 3 & 2 \\ 6 & 4 \end{matrix}$

$= \langle -8, -4, 4\rangle$

unit vector: $\pm \dfrac{1}{|\vec{a} \times \vec{b}|} \langle -8, -4, 4\rangle$

$= \pm \dfrac{1}{\sqrt{64+16+16}} \langle -8, -4, 4\rangle$

$= \pm \dfrac{1}{\sqrt{96}} \langle -8, -4, 4\rangle = \left\langle \dfrac{-8}{\sqrt{96}}, \dfrac{-4}{\sqrt{96}}, \dfrac{4}{\sqrt{96}} \right\rangle$

or

22 [SEE ATTACHED]

$\left\langle \dfrac{8}{\sqrt{96}}, \dfrac{4}{\sqrt{96}}, \dfrac{-4}{\sqrt{96}} \right\rangle$

31. $\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 4 & 3 & -2 \\ 5 & 5 & 1 \end{vmatrix} = i(3+10) - j(4+10) + k(20-$

$\vec{PQ} = \langle 4, 3, -2\rangle$

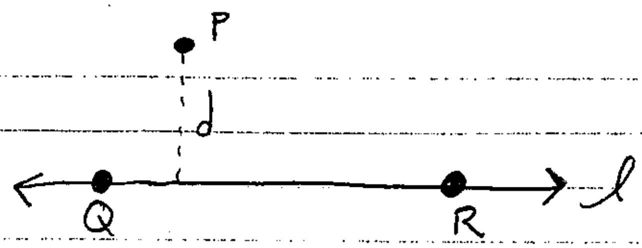
$\vec{PR} = \langle 5, 5, 1\rangle$

$= \langle 13, -14, 5\rangle$ is orthogonal to vectors in Plane.

Area of triangle $= \frac{1}{2} |a \times b| = \frac{1}{2}\sqrt{13^2+14^2+5^2}$

$= \boxed{\frac{1}{2}\sqrt{390}}$

43.



Area of triangle $\Delta PQR = \frac{1}{2}\left|\vec{QP} \times \vec{QR}\right|$

$= \frac{1}{2}\left|\vec{QP}\right|\left|\vec{QR}\right| \sin\theta$ (A

also

$= \frac{1}{2}$ base height

$= \frac{1}{2}\left|\vec{QR}\right| d$ (B)

$$d = \frac{\left|\vec{QP}\right|\left|\vec{QR}\right| \sin\theta}{\left|\vec{QR}\right|}$$

$$= \frac{\left|\vec{QP} \times \vec{QR}\right|}{\left|\vec{QR}\right|}$$

$$= \frac{\left|\vec{a} \times \vec{b}\right|}{\left|\vec{a}\right|}$$

a x+ b+r

45. $(a-b)\times(a+b) \overset{?}{=} 2(a\times b)$

$(a-b)\times(a+b)$

$= (a-b)\times a + (a-b)\times b$

$= a\times a + -b\times a + a\times b + -b\times b$

$= a\times a - (b\times a) + (a\times b) - (b\times b)$

$= (a\times b) + (a\times b)$

$= 2(a\times b)$ ✓

$\boxed{\text{Proof}}$ # 22

$(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = i(a_2b_3 - a_3b_2) - j(a_1b_3 - a_3b_1) + k(a_1b_2 - a_2b_1$$

$$\langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle$$

$$\vec{a} \times \vec{b} \cdot \vec{a} = a_1(a_2b_3 - a_3b_2) + a_2(a_3b_1 - a_1b_3) + a_3(a_1b_1 - a_2b_1$$

$$= a_1a_2b_3 - a_1a_3b_2 + a_2a_3b_1 - a_1a_2b_3 + a_3a_1b_1 - a_3a_2$$

$= 0$ (like terms cancel)

$\boxed{\text{Proof}}$ $\vec{a} \parallel \vec{b}$ iff $\vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow$ $\vec{a}, \vec{b}$ are parallel $\theta = 0, \pi \rightarrow \sin\theta = 0$

$$\therefore |\vec{a}||\vec{b}|\sin\theta = 0$$
$$\therefore |\vec{a} \times \vec{b}| = 0$$
$$\therefore \vec{a} \times \vec{b} = \vec{0}$$

$\Leftarrow$ $a \times b = \vec{0} \rightarrow |a \times b| = 0$

Case 1

$\vec{a}, \vec{b} \neq \vec{0}$ $\quad |\vec{a}||\vec{b}|\sin\theta = 0$

$\therefore \sin\theta = 0$

$\therefore \theta = 0, \pi$ and

hence $\vec{a}, \vec{b}$ are parallel

Case 2

$\vec{a}$ or $\vec{b} = \vec{0}$ $\quad \vec{a} \parallel \vec{b}$

b/c $\vec{0}$ is parallel to all vectors