BC Q502

HW Lesson 1 – solutions

1. a) NO b) Yes c) Yes d) Yes e) NO f) NO 8:00

7. $1(5)-2(0)+3(9) = \underline{32}$

9. $a \cdot b = |a||b|\cos\theta = (6)(5)\cos(2\pi/3) = 30(-1/2) = \underline{-15}$ $\sqrt{100}\ \sqrt{4}\ \sqrt{2}$

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11. $\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta = (1)(1)\cos(60°) = 1/2$

$\vec{u}\cdot\vec{w} = |\vec{u}||\vec{w}|\cos\theta = (1)(1)\cos(120°) = -1/2$ 36 $20\ \sqrt{2}$ 10

tip to tip $120°$

13 b. ✓

20. $\cos\theta = \frac{a\cdot b}{|a|\,|b|} = \frac{10}{\sqrt{9}\sqrt{25}} = \frac{2}{3}$ $\theta = \cos^{-1}\left(\frac{2}{3}\right)$

21. $\angle BAC$

$\cos\theta = \frac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}||\vec{AC}|} = \frac{20}{\sqrt{40}\ \sqrt{20}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\vec{AB} = \langle 2, 6\rangle$ $\vec{AC} = \langle -2, 4\rangle$

$-4+36$ $4+36$ $4+16$

$\theta_1 = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} \to 45°$

$\theta_2 = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} \to 45°$ likewise

$\theta_3 = \cos^{-1}(0) = \frac{\pi}{2} \to 90°$

23. a) $\vec{a}\cdot\vec{b} = -40$ (neither)

b) $\vec{a}\cdot\vec{b} = 0$ (orthogonal)

c) $\vec{a}\cdot\vec{b} = 0$ (orthogonal)

d) $\vec{a}\cdot\vec{b} = -6-45-24$ (not orthogonal)

$\vec{a} = -\frac{2}{3}\vec{b}$ (parallel)

26. $-6b + b^3 + 2b = 0$

$b^3 - 4b = 0$

$b(b+2)(b-2) = 0$ $b = 0, 2, -2$

27. $\langle a,b,c\rangle\cdot\langle 1,1,0\rangle = a+b = 0$ $b-c=0$

$\langle a,b,c\rangle\cdot\langle 1,0,1\rangle = a+c=0$ $b=c$ $a=-b$

$\langle a,-a,-a\rangle$ ex: $\langle -1,1,1\rangle$ UNIT $\pm\frac{1}{\sqrt{3}}\langle -1,1,1\rangle$

37. $\text{Comp}_a b = \frac{a\cdot b}{|a|} = \frac{3+12-6}{\sqrt{49}} = \frac{9}{7}$

$\text{Proj}_a b = \left(\frac{9}{7}\right)\frac{\langle 3,6,-2\rangle}{7} = \frac{\langle 27,54,-18\rangle}{49} = \left\langle \frac{27}{49}, \frac{54}{49}, \frac{-18}{49}\right\rangle$

43. $\text{Comp}_a b = |b|\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \dfrac{3b_1 - b_3}{\sqrt{10}} = 2$

$$3b_1 - b_3 = 2\sqrt{10}$$

$$b_3 = 3b_1 - 2\sqrt{10}$$

$b_1 = b_1 \qquad b_2 = b_2 \qquad b_3 = 3b_1 - 2\sqrt{10}$

ex: $\langle 1, 5, 3 - 2\sqrt{10} \rangle$

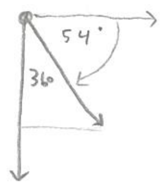
45. Work $= F \cdot D = 48 - 12 + 108 = 144$ N-m

X 47. $\vec{F} = \langle 30\cos 40, 30\sin 40 \rangle$

$\vec{D} = \langle 80, 0 \rangle$

$\vec{F}\cdot\vec{D} = 2400\cos(40)$ ft-lb.

X 48.



$\vec{F} = \langle 400\cos(-54), 400\sin(-54) \rangle$

$\vec{D} = \langle 0, -120 \rangle$

$W = \vec{F}\cdot\vec{D} = -48000\sin(-54) = 48000\sin(54)$

≈ 3883.2 ft-lb

49. SEE ATTACHED

44. (a) $\vec{a}$ and $\vec{b}$ are orthogonal (or) $\vec{a} = \vec{b}$ (or) $|\vec{a}| = |\vec{b}|$

$\dfrac{\vec{a}\cdot\vec{b}}{|a|} = \dfrac{\vec{b}\cdot\vec{a}}{|b|}$ Thus... $\vec{a}$ and $\vec{b}$ are orthogonal or of the same length.

(b) $\vec{a}$ and $\vec{b}$ are orthogonal (or) $\vec{a} = \vec{b}$

$$\frac{\vec{a}}{|\vec{a}|^2} = \frac{\vec{b}}{|\vec{b}|^2}$$

SUPPLEMENT: $\vec{F} = \langle 2, -1, -7 \rangle \qquad D = \langle 1-2,\ 7-(-3),\ 4+5 \rangle$ -4-5

$D = \langle -1, 10, \cancel{1} \rangle$ -9

$\vec{F}\cdot\vec{D} = 2(-1) + (-1)(10) - 7\cancel{(-1)}$ +63

$= -2 - 10 + \cancel{7}$ +63

$= \cancel{\boxed{-5}}$ N-m ← as an example of units

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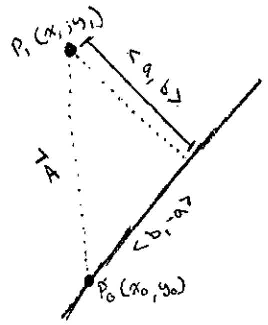

Eqn: $ax + by + c = 0$

$by = -ax - c$

$y = -\frac{a}{b}x - \frac{c}{b}$

Slope is $-\frac{a}{b}$, so $<b, -a>$ can be used

Line that is the distance is perpendicular to $<b, -a>$.

So, this line must be $<a, b>$. Note: b/c $ba + (-ab) = 0$ $\therefore$ ensuring perpendicular or orthogonal

Vector from P_1 to P_0 is $<x_0 - x_1, y_0 - y_1>$

Label this vector $\vec{A}$

$$\text{Comp}_{<a,b>} \vec{A} = \frac{\vec{A} \cdot <a, b>}{|<a, b>|} = \frac{<x_0 - x_1, y_0 - y_1> \cdot <a, b>}{|<a, b>|} = \frac{a(x_0 - x_1) + b(y_0 - y_1)}{\sqrt{a^2 + b^2}}$$

$$= \frac{ax_0 - ax_1 + by_0 - by_1}{\sqrt{a^2 + b^2}}$$

Now a substitution:

$P_0(x_0, y_0)$ is on line $ax + by + c = 0$

so ... $ax_0 + by_0 + c = 0$

$ax_0 + by_0 = -c$

$$= \frac{-ax_1 - by_1 + ax_0 + by_0}{\sqrt{a^2 + b^2}}$$

$$= \frac{-ax_1 - by_1 - c}{\sqrt{a^2 + b^2}} \Longrightarrow \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$\leftarrow$ |abs value| b/c distance is always positive

Distance: $\dfrac{|3 \cdot (-2) + (-4)(3) + 5|}{\sqrt{3^2 + 4^2}} = \dfrac{13}{5}$