Q501. Lesson 2 HW Solutions

[FROM NOTES PACKET]

IV EX 2 $\int \cos^2 x\,dx = \int \frac{1 + \cos 2x}{2}\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$

identity

alt. $= \frac{1}{2}x + \frac{1}{\cancel{4}2}\sin x \cos x + c$

EX 3 $\int \cos^3 x \sin^4 x\,dx = \int \sin^4 x(1-\sin^2 x)\cos x\,dx$

$u = \sin x \quad du = \cos x\,dx$

$\int u^4(1-u^2)\,du = \frac{u^5}{5} - \frac{u^7}{7} + c = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$

V EX 2 $\int \tan^2 x \sec^4 x\,dx = \int \tan^2 x \sec^2 x \sec^2 x\,dx$

$= \int \tan^2 x(1+\tan^2 x)\sec^2 x\,dx$

$u = \tan x \quad du = \sec^2 x$

$= \int u^2(1+u^2)\,du$

$= \frac{u^3}{3} + \frac{u^5}{5} + c = \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + c$

supplemental worksheet Pg. 2

PRINCIPLES OF INTEGRATION (UNIT EXERCISES)

\# 1 (a) $\int x \sin x \, dx$ — Integration by parts — $u = x \quad dv = \sin x \, dx$

(b) $\int \cos x \sin x \, dx$ — u substitution — $u = \sin x$

(c) $\int \tan^7 x \, dx$ — reduction formula

(d) $\int \tan^7 x \sec^2 x \, dx$ — u substitution — $u = \tan x$

(e) $\int 3x^2 / x^3+1 \, dx$ — u substitution — $u = x^3 + 1$

(f) $\int 3x^2 / (x+1)^3 \, dx$ — partial fractions

(g) $\int \tan^{-1} x \, dx$ — Integration by parts — $u = \tan^{-1} x \quad dv = dx$

(h) $\int \sqrt{4-x^2} \, dx$ — trig substitution — $x = 2 \sin \theta$

(i) $\int x\sqrt{4-x^2} \, dx$ — u substitution — $u = 4 - x^2$

\# 2. (a) $\int \sqrt{9+x^2} \, dx$ — $\boxed{x = 3 \tan \theta}$

(b) $\int \sqrt{9-x^2} \, dx$ — $\boxed{x = 3 \sin \theta}$

(c) $\int \sqrt{1-9x^2} \, dx = \int \sqrt{9}\sqrt{1/9 - x^2} \, dx$ — $\boxed{x = \frac{1}{3} \sin \theta}$

(d) $\int \sqrt{x^2 - 9} \, dx$ — $\boxed{x = 3 \sec \theta}$

(e) $\int \sqrt{9+3x^2} \, dx = \int \sqrt{3}\sqrt{3+x^2}$ — $\boxed{x = \sqrt{3} \tan \theta}$

(f) $\int \sqrt{1+(9x)^2} \, dx = \int \sqrt{81}\sqrt{1/81 + x^2}$ — $\boxed{x = \frac{1}{9} \tan \theta}$

Supplemental worksheet pg. 3

7. (a) $\int \sin^4 2x \, dx \qquad u = 2x \qquad du = 2dx$

$$= \frac{1}{2}\int \sin^4 u \, du = \frac{1}{2}\left[\frac{-\sin^3 \cancel{x} \cos \cancel{x}}{4} + \frac{3}{4}\int \sin^2 \cancel{x} \, d\cancel{x}\right]$$

$$= -\frac{1}{8}\sin^3 \cancel{u} \cos \cancel{u} + \frac{3}{8}\int \sin^2 \cancel{u} \, dx$$

$$= -\frac{1}{8}\sin^3 \cancel{u} \cos \cancel{u} + \frac{3}{8}\left[-\frac{\sin \cancel{u} \cos \cancel{u}}{2} + \frac{1}{2}\int d\cancel{u}\right]$$

$$= \boxed{-\frac{1}{8}\sin^3(2x)\cos(2x) - \frac{3}{16}\sin(2x)\cos(2x) + \frac{3}{16}(2x) + C}$$

(b) $\int x \cos^5(x^2) \, du \qquad u = x^2 \qquad du = 2x\,dx$

$$= \frac{1}{2}\int \cos^5 u \, du = \frac{1}{2}\left[\frac{\cos^4 u \sin u}{5} + \frac{4}{5}\int \cos^3 u \, du\right]$$

$$= \frac{1}{10}\cos^4 u \sin u + \frac{4}{10}\int \cos^3 u \, du$$

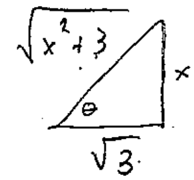
$$= \frac{1}{10}\cos^4 u \sin u + \frac{4}{10}\left[\frac{\cos^2 u \sin u}{3} + \frac{2}{3}\int \cos u \, du\right]$$

$$= \frac{1}{10}\cos^4(x^2)\sin(x^2) + \frac{2}{15}\cos^2 u \sin u \boxed{+\frac{4}{15}\int \cos u \, du} = \boxed{\frac{1}{10}\cos^4(x^2)\sin(x^2) + \frac{2}{15}\cos^2(x^2)\sin(x^2) + \frac{4}{15}\sin(x^2) + C}$$

15. $\int \sqrt{\cos\theta}\,\sin\theta \, d\theta \qquad u = \cos\theta \qquad du = -\sin\theta \, d\theta$

$$= -\int u^{1/2} du = -\frac{2u^{3/2}}{3} + C = \boxed{-\frac{2}{3}(\cos\theta)^{3/2} + C}$$

19. $\int \frac{dx}{(3+x^2)^{3/2}} \qquad$ Let $x = \sqrt{3}\tan\theta$, $dx = \sqrt{3}\sec^2\theta \, d\theta$



$$\sqrt{3}\int \frac{\sec^2\theta \, d\theta}{(3+3\tan^2\theta)^{3/2}} = \sqrt{3}\int \frac{\sec^2\theta \, d\theta}{3^{3/2}(1+\tan^2\theta)^{3/2}}$$

$$= \frac{(3)^{1/2}}{3^{3/2}}\int \frac{\sec^2\theta \, d\theta}{\left[(\sec^2\theta)^{1/2}\right]^3} = \frac{1}{3}\int \frac{1}{\sec\theta} \, d\theta = \frac{1}{3}\int \cos\theta \, d\theta$$

$$= \frac{1}{3}\sin\theta + C$$

$\frac{1}{2} - \frac{3}{2} = -\frac{2}{2} = -1$

$$= \boxed{\frac{1}{3}\frac{x}{\sqrt{x^2+3}} + C}$$

17. $\int x \tan^2(x^2) \sec^2(x^2)\, dx$

$u = x^2 \qquad du = 2x\, dx$

$\frac{1}{2}\int \tan^2(u) \sec^2(u)\, du$

$v = \tan u \qquad dv = \sec^2 u\, du$

$\frac{1}{2}\int v^2 dv = \frac{v^3}{6} + c = \frac{\tan^3 u}{6} + c$

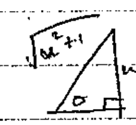
$= \frac{\tan^3(x^2)}{6} + c$

21 $\int \frac{x+3}{\sqrt{x^2+2x+2}}\, dx \qquad x^2+2x+2 = x^2+2x+1+1$

$= (x+1)^2 + 1$

$u = x+1 \qquad du = dx \qquad x = u-1$

$\int \frac{u-1+3}{\sqrt{u^2+1}}\, du = \int \frac{u+2}{\sqrt{u^2+1}}\, du$



$* \int \frac{2}{\sqrt{u^2+1}}\, du$

$= \int \frac{u}{\sqrt{u^2+1}}\, du + \int \frac{2}{\sqrt{u^2+1}}\, du \qquad u = \tan\theta \quad du = \sec^2\theta\, d\theta$

$= 2\int \frac{\sec^2\theta\, d\theta}{\sqrt{\tan^2\theta + 1}}$

$v = u^2+1$

$dv = 2u\, du$

$= 2\int \frac{\sec^2\theta\, d\theta}{\sec\theta}$

$= \frac{1}{2}\int v^{-1/2} dv + \int \frac{2}{\sqrt{u^2+1}}\, du \qquad = 2\int \sec\theta$

$= 2 \ln|\sec\theta + \tan\theta| + c$

$= \frac{1}{2}\cdot\frac{2}{1} v^{1/2} + \int \frac{2}{\sqrt{u^2+1}}\, du \qquad = 2\ln\left|\sqrt{u^2+1} + u\right| + c$

$= \sqrt{u^2+1} + \int \frac{2}{\sqrt{u^2+1}}\, du \qquad = 2\ln\left|\sqrt{x^2+2x+2} + x+1\right| + c$

$= \sqrt{x^2+2x+2} + 2\ln\left|\sqrt{x^2+2x+2} + x+1\right| + c$

(23) $\displaystyle\int \frac{dx}{(x-1)(x+2)(x-3)}$

$$\int\left(\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x-3}\right)dx = \frac{-1}{6}\ln|x-1| + \frac{1}{15}\ln|x+2| + \frac{1}{10}\ln|x-3|$$

$$A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2) = 1$$
$$A(x^2-x-6) + B(x^2-4x+3) + C(x^2+x-2) = 1$$

$A + B + C = 0$ I

$-A - 4B + C = 0$ II

$-6A + 3B - 2C = 1$ III

$-A - B - C = 0$
$\underline{-A - 4B + C = 0}$
$-2A - 5B = 0$

$-2A - 8B + 2C = 0$
$\underline{-6A + 3B - 2C = 1}$
$-8A - 5B = 1$

* "Pick a value" strategy works quicker here!

$-2A - 5B = 0$
$\underline{+8A + 5B = -1}$
$6A = -1$
$A = -1/6$

$-2(-1/6) - 5B = 0$
$\frac{1}{3} - 5B = 0$
$-5B = -\frac{1}{3}$
$B = +\frac{1}{15}$

$\frac{1}{15} - \frac{1}{6} + C = 0$

$\frac{2}{30} - \frac{5}{30} = -C$

$\frac{-3}{30} = -C$

$\frac{1}{10} = C$

extra: $\displaystyle\int \frac{dx}{x^2-4x+8}$

$$x^2-4x+8 = x^2-4x+4+4$$
$$= (x-2)^2+4$$

$u = x-2 \qquad du = dx \qquad x = u+2$

let $v = \frac{u}{2}$ $\quad dv = \frac{1}{2}\,du$

$$\int \frac{du}{u^2+4} = \frac{1}{4}\int \frac{du}{\left(\frac{u}{2}\right)^2+1} = \frac{1}{4}\cdot 2\tan^{-1}\left(\frac{u}{2}\right)+c$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)+c$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x-2}{2}\right)+c$$