11. $t^3 + t^2 - 2t = t(t^2 + t - 2) = t(t+2)(t-1)$

$$\frac{1}{t^3+t^2-2t} = \frac{A}{t} + \frac{B}{t+2} + \frac{C}{t-1}$$

$$1 = A(t+2)(t-1) + B(t)(t-1) + C(t)(t+2)$$

$$= A(t^2+t-2) + B(t^2-t) + C(t^2+2t)$$

$$= (A+B+C)t^2 + (A-B+2C)t - 2A$$

Equating coefficients of like terms gives

$A + B + C = 0, A - B + 2C = 0$, and $-2A = 1$.

Solving the system simultaneously yields

$A = -\frac{1}{2}, B = \frac{1}{6}, C = \frac{1}{3}$.

$$\int \frac{dt}{t^3+t^2-2t} = \int \frac{-1/2}{t}\,dt + \int \frac{1/6}{t+2}\,dt + \int \frac{1/3}{t-1}\,dt$$

$$= -\frac{1}{2}\ln|t| + \frac{1}{6}\ln|t+2| + \frac{1}{3}\ln|t-1| + C$$

12. $2t^3 - 8t = 2t(t^2 - 4) = 2t(t-2)(t+2)$

$$\frac{t+3}{2t^3-8t} = \frac{A}{t} + \frac{B}{t-2} + \frac{C}{t+2}$$

$$t + 3 = 2A(t-2)(t+2) + 2B(t)(t+2) + 2C(t)(t-2)$$

$$= 2A(t^2-4) + 2B(t^2+2t) + 2C(t^2-2t)$$

$$= (2A+2B+2C)t^2 + (4B-4C)t - 8A$$

Equating coefficients of like terms gives

$2A + 2B + 2C = 0, 4B - 4C = 1, -8A = 3$

Solving the system simultaneously yields

$A = -\frac{3}{8}, B = \frac{5}{16}, C = \frac{1}{16}$.

$$\int \frac{t+3}{2t^3-8t}\,dt = \int \frac{-3/8}{t}\,dt + \int \frac{5/16}{t-2}\,dt + \int \frac{1/16}{t+2}\,dt$$

$$= -\frac{3}{8}\ln|t| + \frac{5}{16}\ln|t-2| + \frac{1}{16}\ln|t+2| + C$$

13.

$$\begin{array}{r} s \\ s^2+4 \overline{)\, s^3 \qquad\quad} \\ \underline{s^3 \;+ 4s} \\ -4s \end{array}$$

$$\frac{s^3}{s^2+4} = s + \frac{-4s}{s^2+4}$$

$$\int \frac{s^3}{s^2+4}\,ds = \int s\,ds - \int \frac{4s}{s^2+4}\,ds$$

$$= \frac{1}{2}s^2 - 2\ln(s^2+4) + C$$

14.

$$\begin{array}{r} s^2 - 1 \\ s^2+1 \overline{)\, s^4 \qquad\qquad + 2s} \\ \underline{s^4 \qquad + s^2 \qquad\;\;} \\ -s^2 + 2s \\ \underline{-s^2 \qquad - 1} \\ 2s + 1 \end{array}$$

$$\frac{s^4+2s}{s^2+1} = s^2 - 1 + \frac{2s+1}{s^2+1} = s^2 - 1 + \frac{2s}{s^2+1} + \frac{1}{s^2+1}$$

$$\int \frac{s^4+2s}{s^2+1}\,ds = \int (s^2-1)\,ds + \int \frac{2s\,ds}{s^2+1} + \int \frac{ds}{s^2+1}$$

$$= \frac{1}{3}s^3 - s + \ln(s^2+1) + \tan^{-1} s + C$$

15.

$$\begin{array}{r} 5 \\ x^2+x+1 \overline{)\, 5x^2 \qquad\qquad} \\ \underline{5x^2 + 5x + 5} \\ -5x - 5 \end{array}$$

$$\frac{5x^2}{x^2+x+1} = 5 - \frac{5x+5}{x^2+x+1}$$

$$\int \frac{5x^2\,dx}{x^2+x+1} = \int 5\,dx - 5\int \frac{x+1}{x^2+x+1}\,dx$$

$$= 5x - 5\int \frac{x+1}{x^2+x+1}\,dx$$

To evaluate the second integral, complete the square in the denominator.

$$x^2 + x + 1 = x^2 + x + \frac{1}{4} + \frac{3}{4} = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$\int \frac{x+1}{x^2+x+1}\,dx$$

$$= \int \frac{x+1}{(x+1/2)^2 + 3/4}\,dx$$

$$= \int \frac{x+1/2}{(x+1/2)^2+3/4}\,dx + \int \frac{1/2}{(x+1/2)^2+3/4}\,dx$$

$$= \frac{1}{2}\ln\left[\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}\right] + \frac{1}{2}\int \frac{dx}{(x+1/2)^2 + (\sqrt{3}/2)^2}$$

$$= \frac{1}{2}\ln(x^2+x+1) + \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)\tan^{-1}\left(\frac{x+1/2}{\sqrt{3}/2}\right)$$

$$= \frac{1}{2}\ln(x^2+x+1) + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

The second integral was evaluated by using Formula 16 from the Brief Table of Integrals.

$$\int \frac{5x^2\,dx}{x^2+x+1}$$

$$= 5x - \frac{5}{2}\ln(x^2+x+1) - \frac{5}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$$

16. $(x-1)(x^2+2x+1) = (x-1)(x+1)^2$

$$\frac{x^2}{(x-1)(x^2+2x+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$$

$$= A(x^2+2x+1) + B(x^2-1) + C(x-1)$$

$$= (A+B)x^2 + (2A+C)x + A - B - C$$

Equating coefficients of like terms gives

$A + B = 1, 2A + C = 0$, and $A - B - C = 0$.

Solving the system simultaneously yields

$A = \frac{1}{4}, B = \frac{3}{4}, C = -\frac{1}{2}$.

$$\int \frac{x^2\,dx}{(x-1)(x^2+2x+1)}$$

$$= \int \frac{1/4}{x-1}\,dx + \int \frac{3/4}{x+1}\,dx + \int \frac{-1/2}{(x+1)^2}\,dx$$

$$= \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2(x+1)} + C$$

Lesson 1 Hw Solutions [TOTT]

17. $(x^2-1)^2 = (x+1)^2(x-1)^2$

$$\frac{1}{(x^2-1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$1 = A(x+1)(x-1)^2 + B(x-1)^2 + C(x+1)^2(x-1) + D(x+1)^2$$

$$= A(x^3 - x^2 - x + 1) + B(x^2 - 2x + 1) + C(x^3 + x^2 - x - 1) + D(x^2 + 2x + 1)$$

$$= (A + C)x^3 + (-A + B + C + D)x^2 + (-A - 2B - C + 2D)x + (A + B - C + D)$$

Equating coefficients of like terms gives

$A + C = 0$, $-A + B + C + D = 0$,

$-A - 2B - C + 2D = 0$, and $A + B - C + D = 1$

Solving the system simultaneously yields

$A = \frac{1}{4}, B = \frac{1}{4}, C = -\frac{1}{4}, D = \frac{1}{4}$.

$$\int \frac{dx}{(x^2-1)^2} = \int \frac{1/4}{x+1}\,dx + \int \frac{1/4}{(x+1)^2}\,dx + \int \frac{-1/4}{x-1}\,dx + \int \frac{1/4}{(x-1)^2}\,dx$$

$$= \frac{1}{4}\ln|x+1| - \frac{1}{4(x+1)} - \frac{1}{4}\ln|x-1| - \frac{1}{4(x-1)} + C$$

18. $x^2 + 5x - 6 = (x+6)(x-1)$

$$\frac{x+4}{x^2+5x-6} = \frac{A}{x+6} + \frac{B}{x-1}$$

$$x + 4 = A(x-1) + B(x+6)$$

$$= (A + B)x + (-A + 6B)$$

Equating coefficients of like terms gives

$A + B = 1$ and $-A + 6B = 4$.

Solving the system simultaneously yields

$A = \frac{2}{7}, B = \frac{5}{7}$.

$$\int \frac{x+4}{x^2+5x-6}\,dx = \int \frac{2/7}{x+6}\,dx + \int \frac{5/7}{x-1}\,dx$$

$$= \frac{2}{7}\ln|x+6| + \frac{5}{7}\ln|x-1| + C$$

19. Complete the square in the denominator.

$$r^2 - 2r + 2 = r^2 - 2r + 1 + 1 = (r-1)^2 + 1$$

$$\int \frac{2\,dr}{r^2 - 2r + 2} = \int \frac{2\,dr}{(r-1)^2 + 1} = 2\tan^{-1}(r-1) + C$$

20. Complete the square in the denominator.

$$r^2 - 4r + 5 = r^2 - 4r + 4 + 1 = (r-2)^2 + 1$$

$$\int \frac{3\,dr}{r^2 - 4r + 5} = \int \frac{3\,dr}{(r-2)^2 + 1} = 3\tan^{-1}(r-2) + C$$

21. $x^3 - 1 = (x-1)(x^2 + x + 1)$

$$\frac{x^2 - 2x - 2}{x^3 - 1} = \frac{A}{x-1} + \frac{Bx + C}{x^2 + x + 1}$$

$$x^2 - 2x - 2 = A(x^2 + x + 1) + (Bx + C)(x - 1)$$

$$= (A + B)x^2 + (A - B + C)x + (A - C)$$

Equating coefficients of like terms gives

$A + B = 1$, $A - B + C = -2$, and $A - C = -2$.

Solving the system simultaneously yields

$A = -1, B = 2, C = 1$.

$$\int \frac{x^2 - 2x - 2}{x^3 - 1}\,dx = \int \frac{-1}{x-1}\,dx + \int \frac{2x+1}{x^2+x+1}\,dx$$

$$= -\ln|x-1| + \ln(x^2 + x + 1) + C$$

22. $x^3 + 1 = (x+1)(x^2 - x + 1)$

$$\frac{x^2 - 4x + 4}{x^3 + 1} = \frac{A}{x+1} + \frac{Bx + C}{x^2 - x + 1}$$

$$x^2 - 4x + 4 = A(x^2 - x + 1) + (Bx + C)(x + 1)$$

$$= (A + B)x^2 + (-A + B + C)x + (A + C)$$

Equating coefficients of like terms gives

$A + B = 1$, $-A + B + C = -4$, and $A + C = 4$.

Solving the system simultaneously yields

$A = 3, B = -2, C = 1$.

$$\int \frac{x^2 - 4x + 4}{x^3 + 1}\,dx = \int \frac{3}{x+1}\,dx + \int \frac{-2x+1}{x^2 - x + 1}\,dx$$

$$= 3\ln|x+1| - \ln(x^2 - x + 1) + C$$

23. $$\frac{3x^2 - 2x + 12}{(x^2+4)^2} = \frac{Ax + B}{x^2 + 4} + \frac{Cx + D}{(x^2+4)^2}$$

$$3x^2 - 2x + 12 = (Ax + B)(x^2 + 4) + (Cx + D)$$

$$= Ax^3 + Bx^2 + (4A + C)x + 4B + D$$

Equating coefficients of like terms gives

$A = 0$, $B = 3$, $4A + C = -2$, and $4B + D = 12$

Solving the system simultaneously yields

$A = 0, B = 3, C = -2, D = 0$.

$$\int \frac{3x^2 - 2x + 12}{(x^2+4)^2}\,dx = \int \frac{3}{x^2+4}\,dx + \int \frac{-2x}{(x^2+4)^2}\,dx$$

$$= \frac{3}{2}\tan^{-1}\frac{x}{2} + \frac{1}{x^2+4} + C$$

The first integral was evaluated by using Formula 16 from the Brief Table of Integrals.

Lesson 1

$$+\frac{3}{4}\ln|x+1| + \frac{1}{2}\frac{1}{(x+1)} + \frac{1}{4}\ln|x-1| +$$

16] $$\frac{x^2}{(x-1)(x^2+2x+1)} = \frac{x^2}{(x-1)(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{x-1}$$

$$A(x+1)(x-1) + B(x-1) + C(x+1)^2 = A(x^2-1) + B(x-1) + C(x^2+2x+1)$$

$$(A+C)x^2 + (B+2C)x - A - B + C = x^2$$

$A + C = 1$
$B + 2C = 0$
$-A - B + C = 0$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ -1 & -1 & 1 & 0 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & 2 & 1 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 4 & 1 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1/4 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 1 & 1/4 \end{array}\right]$$

17] $$\frac{1}{(x^2-1)^2} = \frac{1}{(x+1)^2(x-1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$= \frac{1}{4(x+1)} + \frac{1}{4(x+1)^2} - \frac{1}{4(x-1)} + \frac{1}{4(x-1)^2}$$

$$A(x+1)(x-1)^2 + B(x-1)^2 + C(x+1)^2(x-1) + D(x+1)^2$$

$$\frac{1}{4}\ln|x+1| - \frac{1}{4}\frac{1}{(x+1)} - \frac{1}{4}\ln|x-1| - \frac{1}{4}\frac{1}{(x-1)}$$

$$A(x+1)(x^2-2x+1) + B(x^2-2x+1) + C(x^2+2x+1)(x-1) + D(x^2+2x+1)$$

$$A(x^3 - 2x^2 + x + x^2 - 2x + 1) + B(\quad) + C(x^3 + 2x^2 + x - x^2 - 2x - 1) + D(\quad)$$

$$A(x^3 - x^2 - x + 1) + B(x^2 - 2x + 1) + C(x^3 + x^2 - x - 1) + D(x^2 + 2x + 1)$$

21] $$\frac{x^2-2x-2}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} = \frac{-1}{x-1} + \frac{2x+1}{x^2+x+1}$$

$$= -\ln|x-1| + \ln(x^2+x+1) + C$$

$$A(x^2+x+1) + (Bx+C)(x-1)$$

$$Ax^2 + Ax + A + Bx^2 - Bx + Cx - C = x^2 - 2x - 2$$

$$A + B = 1 \quad A - B + C = -2 \quad A - C = -2$$

(*) 22] $$\frac{x^2-4x+4}{x^3+1} = \frac{A}{(x+1)} + \frac{Bx+C}{x^2-x+1} = \boxed{\frac{3}{x+1} + \frac{1-2x}{x^2-x+1}}$$

$$= 3\ln|x+1| - \ln(x^2-x+1) + C$$

$$A(x^2-x+1) + (Bx+C)(x+1)$$

$$Ax^2 - Ax + A + Bx^2 + Bx + Cx + C = x^2 - 4x + 4$$

$A + B = 1$
$-A + B + C = -4$
$A + C = 4$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ -1 & 1 & 1 & -4 \\ 1 & 0 & 1 & 4 \end{array}\right] \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & -3 \\ 0 & 1 & -1 & -3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 1/2 & -3/2 \\ 0 & 1 & -1 & -3 \end{array}\right] \quad \begin{array}{cccc} 1 & 0 & -1/2 & 5/2 \\ 0 & 1 & 1/2 & -3/2 \\ 0 & 0 & -3/2 & -3/2 \end{array} \quad \begin{array}{cccc} 1 & 0 & -1/2 & 5/2 \\ 0 & 1 & 1/2 & -3/2 \\ 0 & 0 & 1 & 1 \end{array}$$

(*) 23]

$$\frac{3x^2-2x+12}{(x^2+4)^2} = \frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2}$$

$$(Ax+B)(x^2+4) + (Cx+D)$$

$$Ax^3 + 4Ax + Bx^2 + 4B + Cx + D$$

$$Ax^3 + Bx^2 + (4A+C)x + 4B+D = 3x^2 - 2x + 12$$

$A = 0$
$B = 3$
$C = -2$
$D = 0$

$$\frac{3}{x^2+4} - \frac{2x}{(x^2+4)^2}$$

$$\frac{3}{4} \cdot \frac{1}{1+\left(\frac{x}{2}\right)^2} - \frac{2x}{(x^2+4)^2}$$

$u = x^2+4$
$du = 2x\,dx$
$dx = \frac{du}{2x}$

$\int u^{-2}\,d$

$+u^{-1}$

$$\boxed{\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{(x^2+4)^{\times}} + C}$$

Lesson 1

21) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & -1 & 1 & -2 \\ 1 & 0 & -1 & -2 \end{array}\right]$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & -3 \\ 0 & -1 & -1 & -3 \end{array}\right]$ $-\frac{6}{2} + \frac{3}{2}$

$\left[\begin{array}{cccc} 1 & 1 & 0 & 1 \\ 0 & 1 & -1/2 & 3/2 \\ 0 & -1 & -1 & -3 \end{array}\right]$ $\begin{array}{cccc} 1 & 0 & 1/2 & -1/2 \\ 0 & 1 & -1/2 & 3/2 \\ 0 & 0 & -3/2 & -3/2 \end{array}$

$\begin{array}{cccc} 1 & 0 & 1/2 & -1/2 \\ 0 & 1 & -1/2 & 3/2 \\ 0 & 0 & 1 & 1 \end{array}$ $-1/2$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{array}\right]$

17) $(A+C)x^3 + (B+C+D)x^2 + (-2B-C+2D)x + A+B-C+D$

$A+C=0$
$B+C+D=0$
$-2B-C+2D=0$
$A+B-C+D=1$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & 2 & 0 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right]$ $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & 2 & 0 \\ 0 & 1 & -2 & 1 & 1 \end{array}\right]$

$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 3 & 0 & -1 \end{array}\right]$ $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & -1/3 \\ 0 & 0 & 1 & 4 & 0 \end{array}\right]$ $\begin{array}{ccccc} 1 & 0 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 1 & 4/3 \\ 0 & 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 4 & 1/3 \end{array}$

$\begin{array}{ccccc} 1 & 0 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 1 & 4/3 \\ 0 & 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 1 & 1/7 \end{array}$ $\begin{array}{ccccc} 1 & 0 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 0 & 25/21 \\ 0 & 0 & 1 & 0 & -1/3 \\ 0 & 0 & 0 & 1 & 1/7 \end{array}$

$4/3 - 1/7$

$\frac{28}{21} - \frac{3}{21}$

25

(17) $(A+C)x^3 + (-A+B+C+D)x^2$
$(-A-2B-C+2D)x + A + B - C + D$

$A + C = 0$
$-A + B + C + D = 0$
$-A - 2B - C + 2D = 0$
$A + B - C + D = 1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 \\ -1 & -2 & -1 & 2 & 0 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right] \quad \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & -2 & 0 & 2 & 0 \\ 0 & 1 & -2 & 1 & 1 \end{array}\right] \quad \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 4 & 4 & 0 \\ 0 & 0 & 4 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1/4 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] \quad \begin{array}{ccccc} 1 & 0 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & 1 & 1/2 \\ 0 & 0 & 1 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 1/4 \end{array} \quad \begin{array}{ccccc} 1 & 0 & 0 & 0 & 1/4 \\ 0 & 1 & 0 & 0 & 1/4 \\ 0 & 0 & 1 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 1/4 \end{array}$$