10.3 # 41 $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{(r\sin\theta)'}{(r\cos\theta)'} = \frac{((2-3\sin\theta)(\sin\theta))'}{((2-3\sin\theta)(\cos\theta))'}$

$$= \frac{(2-3\sin\theta)\cos\theta + \sin\theta(-3\cos\theta)}{(2-3\sin\theta)(-\sin\theta) + \cos\theta(-3\cos\theta)}$$

$\theta = 0$: $dy/dx = -2/3$

$\theta = \pi/2$: $dy/dx = 0$

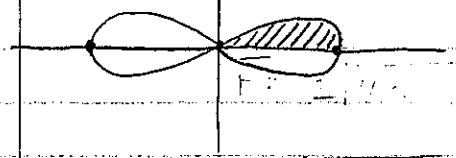
$\theta = \pi$: $dy/dx = 2/3$

$\theta = 3\pi/2$: $dy/dx = 0$

10.3 # 46 $r = 2\sin(4\theta)$

$$A = \int_0^{2\pi} \frac{1}{2}(2\sin(4\theta))^2 d\theta = \boxed{2\pi}$$

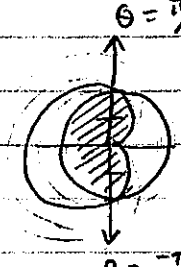
10.3 # 47 $r = \sqrt{4\cos(2\theta)}$



θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π
r	2	0	DNE	DNE	2

$$A = 2\int_0^{\pi/4} \frac{1}{2}(4\cos(2\theta))\, d\theta = \boxed{2}$$

10.3 # 53



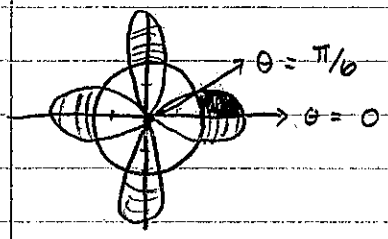
$$A = \int_{-\pi/2}^{\pi/2} \frac{1}{2}(2-2\cos\theta)^2 d\theta + \int_{\pi/2}^{3\pi/2} \frac{1}{2}(2)^2 d\theta = 3\pi - 8 + \frac{\pi(2)^2}{2}$$

($\pi/2$ to $3\pi/2$: half circle)

$$= \boxed{5\pi - 8}$$

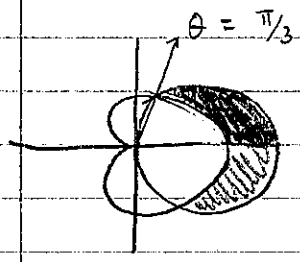
10.3 # 56

$$8\int_0^{\pi/6}\left[\frac{1}{2}(4\cos 2\theta)^2-\frac{1}{2}(2)^2\right]d\theta = 4\sqrt{3}+\frac{8\pi}{3}$$



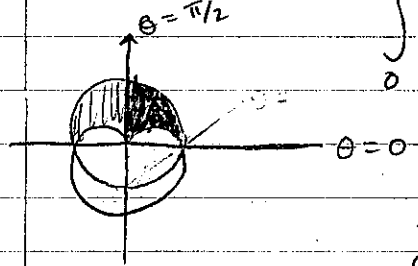
10.3 # 57

$$2\int_0^{\pi/3}\frac{1}{2}\left[(3\cos\theta)^2-(1+\cos\theta)^2\right]d\theta = \pi$$



10.3 # 58

$$\int_0^{\pi}\frac{1}{2}\left[(2)^2-(2-2\sin\theta)^2\right]d\theta = 4.858$$



10.3 # 59

$$A = \int_0^{\pi}\frac{1}{2}\left[2\sin 3\theta\right]^2 d\theta = \pi$$

$$\frac{dy}{dx} = \frac{6\sin\theta\cos 3\theta + 2\cos\theta\sin 3\theta}{6\cos\theta\cos 3\theta - 2\sin\theta\sin 3\theta}\Bigg|_{\theta=\pi/4} = \frac{1}{2}$$

Section 10.3 Polar Functions (pp. 548–559)

Exploration 1 Graphing Polar Curves Parametrically

See text page 551.

Quick Review 10.3

1. $x = 4\ \cos 30 = 2\sqrt{3}$
   $y = 4\ \sin 30 = 2$
   $\langle 2\sqrt{3},\ 2\rangle$

2. $A = \pi r^2 \dfrac{30}{360} = \pi(6)^2 \dfrac{30}{360} = 3\pi$

3. $A = \pi r^2\ \dfrac{\pi}{8}\left(\dfrac{1}{2\pi}\right) = \dfrac{1}{16}\pi\ (8)^2 = 4\pi$

4. $x^2 + y^2 = 25$

5. Graph $y = \left(\dfrac{4-x^2}{3}\right)^{1/2}$ and $y = -\left(\dfrac{4-x^2}{3}\right)^{1/2}$

6. $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{d/dy(5\sin t)}{d/dx(3\cos t)} = \dfrac{5\cos t}{-3\sin t} = -\dfrac{5}{3}\cot t$

7. $-\dfrac{5}{3}\cot(2) = 0.763$

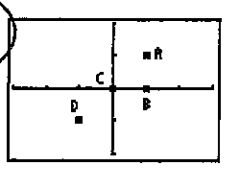
8. $x = 3\cos t = 0$
   $t = \dfrac{\pi}{2}$
   $y = 5\sin t$
   $y = 5\sin\left(\dfrac{\pi}{2}\right) = 5(\pm 1)$
   $y = \pm 5$
   $(0, -5)$ and $(0, 5)$

9. $y = 5\sin t = 0$
   $t = 0$
   $x = 3\cos t$
   $x = 3\cos(0) = 3(\pm 1)$
   $x = \pm 3$
   $(-3, 0)$ and $(3, 0)$

10. $\int_0^{\pi} \sqrt{(3\cos t)^2 + (5\sin t)^2}\, dt \approx 12.763$

Section 10.3 Exercises

1.



[–3, 3] by [–2, 2]

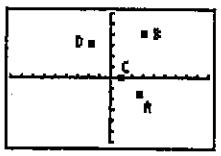
(a) $x = \sqrt{2}\cos\left(\dfrac{\pi}{4}\right) = 1$
$y = \sqrt{2}\sin\left(\dfrac{\pi}{4}\right) = 1$
$(1,\ 1)$

(b) $x = 1\cos(0) = 1$
$y = 1\sin(0) = 0$
$(1, 0)$

(c) $x = 0\cos\left(\dfrac{\pi}{2}\right) = 0\text{w}$
$y = 0\sin\left(\dfrac{\pi}{2}\right) = 0$
$(0,\ 0)$

(d) $x = -\sqrt{2}\cos\left(\dfrac{\pi}{4}\right) = -1$
$y = -\sqrt{2}\sin\left(\dfrac{\pi}{4}\right) = -1$
$(-1, -1)$

2.



[–9, 9] by [–6, 6]

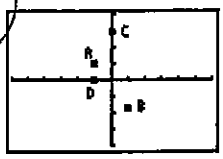
(a) $x = -3\cos\left(\dfrac{5\pi}{6}\right) = \dfrac{3\sqrt{3}}{2}$
$y = -3\sin\left(\dfrac{5\pi}{6}\right) = \dfrac{-3}{2}$
$\left(\dfrac{3\sqrt{3}}{2}, \dfrac{-3}{2}\right)$

(b) $x = 5\cos\left(\tan^{-1}\left(\dfrac{4}{3}\right)\right) = 3$
$y = 5\sin\left(\tan^{-1}\left(\dfrac{4}{3}\right)\right) = 4$
$(3, 4)$

(c) $x = -1\cos 7\pi = 1$
$y = -1\sin 7\pi = 0$
$(1, 0)$

(d) $x = 2\sqrt{3}\cos\left(\dfrac{2\pi}{3}\right) = -\sqrt{3}$
$y = 2\sqrt{3}\sin\left(\dfrac{2\pi}{3}\right) = 3$
$\left(-\sqrt{3}, 3\right)$

3.



[–6, 6] by [–4, 4]

(a) $r=\sqrt{-1^2+1^2}=\sqrt{2}$

$\theta=\tan^{-1}\left(\frac{1}{-1}\right)=-\frac{5\pi}{4},\frac{3\pi}{4}$

$\left(\sqrt{2},\frac{-5\pi}{4}\right)$ and $\left(\sqrt{2},\frac{3\pi}{4}\right)$

(b) $r=\sqrt{1+\left(-\sqrt{3}\right)^2}=\pm 2$

$\theta=\tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)=-\frac{\pi}{3},\frac{2\pi}{3}$

$\left(-2,\frac{2\pi}{3}\right)$ and $\left(2,-\frac{\pi}{3}\right)$

(c) $r=\sqrt{0^2+3^2}=\pm 3$

$\theta=\tan^{-1}\left(\frac{3}{0}\right)=\frac{\pi}{2},\frac{5\pi}{2}$

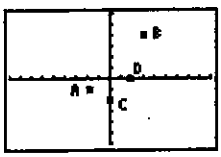
$\left(3,\frac{\pi}{2}\right)$ and $\left(3,\frac{5\pi}{2}\right)$

(d) $r=\sqrt{(-1)^2+0^2}=\pm 1$

$\theta=\tan^{-1}\left(\frac{0}{1}\right)=0,\pi$

$(-1,0)$ and $(1,\pi)$

4.



[–9, 9] by [–6, 6]

(a) $r=\sqrt{\left(-\sqrt{3}\right)^2+(-1)^2}=\pm 2$

$\theta=\tan^{-1}\left(\frac{-1}{-\sqrt{3}}\right)=\frac{\pi}{6},\frac{7\pi}{6}$

$\left(-2,\frac{\pi}{6}\right)$ and $\left(2,\frac{7\pi}{6}\right)$

(b) $r=\sqrt{3^2+4^2}=\pm 5$

$\theta=\tan^{-1}\left(\frac{4}{3}\right)$

$\left(-5,\pi+\tan^{-1}\left(\frac{4}{3}\right)\right)$ and $\left(5,\tan^{-1}\left(\frac{4}{3}\right)\right)$

(c) $r=\sqrt{0^2+(-2)^2}=\pm 2$

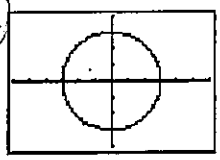
$\theta=\tan^{-1}\left(\frac{-2}{0}\right)=-\frac{\pi}{2},\frac{3\pi}{2}$, $\left(2,-\frac{\pi}{2}\right)$ and $\left(2,\frac{3\pi}{2}\right)$

(d) $r=\sqrt{2^2+0^2}=\pm 2$

$\theta=\tan^{-1}\left(\frac{0}{2}\right)=0,2\pi$

$(2,0)$ and $(2,2\pi)$

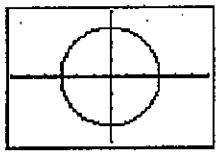
5.



[–6, 6] by [–4, 4]

$0\le\theta\le 2\pi$

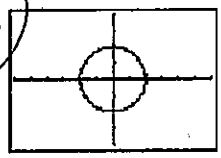
6.



[–6, 6] by [–4, 4]

$0\le\theta\le 2\pi$

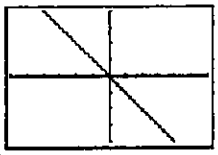
7.



[–6, 6] by [–4, 4]

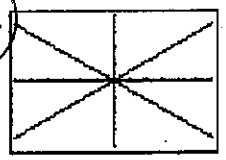
$0\le\theta\le 2\pi$

8.



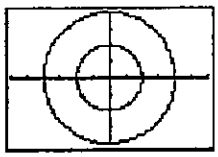
[–6, 6] by [–4, 4]

9.



[–6, 6] by [–4, 4]

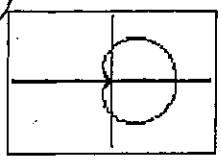
10.



[–6, 6] by [–4, 4]

$0\le\theta\le 2\pi$

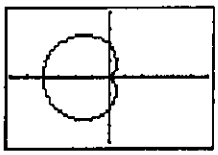
11. Cardioid



[–3, 3] by [–2, 2]

$0 \le \theta \le 2\pi$

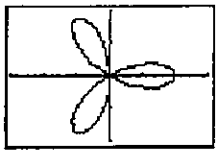
12. Cardioid



[–6, 6] by [–4, 4]

$0 \le \theta \le 2\pi$

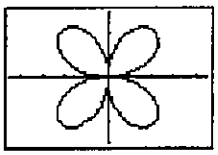
13. Rose



[–3, 3] by [–2, 2]

$0 \le \theta \le \pi$

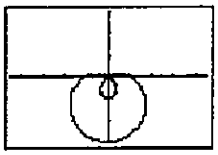
14. Rose



[–4.5, 4.5] by [–3, 3]

$0 \le \theta \le 2\pi$

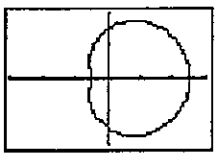
15. Limaçon



[–4.5, 4.5] by [–3, 3]

$0 \le \theta \le 2\pi$

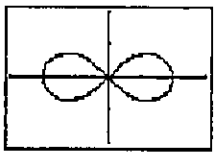
16. Limaçon



[–3, 3] by [–2, 2]

$0 \le \theta \le 2\pi$

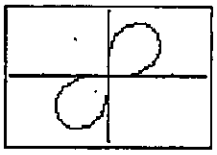
17. Lemniscate



[–3, 3] by [–2, 2]

$-\pi/4 \le \theta \le \pi/4$

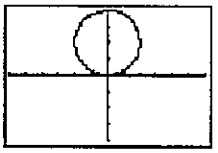
18. Lemniscate



[–1.5, 1.5] by [–1, 1]

$0 \le \theta \le \pi/2$

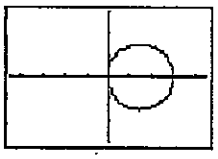
19. Circle



[–6, 6] by [–4, 4]

$0 \le \theta \le \pi$

20. Circle



[–4.5, 4.5] by [–3, 3]

$0 \le \theta \le \pi$

21. $r = 4\csc\theta$

$$1 = \frac{4}{r\sin\theta}$$

$y = 4$, a horizontal line

22. $r = -3\sec\theta$

$$1 = \frac{-3}{r\cos\theta}$$

$x = -3$, a verticle line

23. $x + y = 1$, a line

(slope = –1, y-intercept = 1)

24. $r^2 = x^2 + y^2 = 1$,

a circle (center = (0, 0), radius = 1)

25. $r = \dfrac{5}{\sin\theta - 2\cos\theta}$

$1 = \dfrac{5}{r\sin\theta - 2r\cos\theta}$

$y - 2x = 5$, a line (slope = 2, y-intercept = 5)

26. $r^2 \sin 2\theta = 2$

$r^2(2\sin\theta\cos\theta) = 2$

$2xy = 2$

$xy = 1$, a hyperbola

27. $\pi^2\cos^2\theta = r^2\sin\theta$

$x^2 = y^2$, the union of two lines: $y = \pm x$

28. $r^2 = -4r\cos\theta$

$x^2 + y^2 = -4x$

$x^2 + 4x + 4 - 4 + y^2 = 0$

$(x+2)^2 + y^2 = 4$, a circle (center = (−2, 0), radius = 2)

29. $r^2 = 8r\sin\theta$

$x^2 + y^2 - 8r\sin\theta + 16 - 16 = 0$

$x^2 + (y-4)^2 = 16$, a circle (center = (−2, 0), radius = 2)

30. $r = 2\cos\theta + 2\sin\theta$

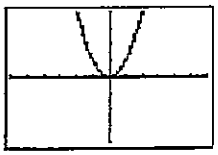
$r^2 = 2r\cos\theta + 2r\sin\theta$

$x^2 + y^2 = 2x + 2y$

$(x-1)^2 + (y-1)^2 = 2$,

a circle (center = (1, 1), radius = $\sqrt{2}$)

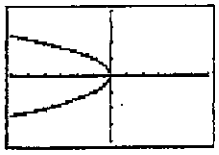
31. It is a parabola.



[−6, 6] by [−4, 4]

$0 \le \theta \le 2\pi$

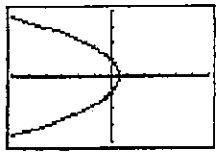
32. It is a parabola.



[−6, 6] by [−4, 4]

$0 \le \theta \le 2\pi$

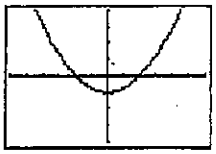
33. It is a parabola.



[−6, 6] by [−4, 4]

$0 \le \theta \le 2\pi$

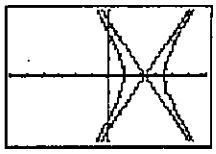
34. It is a parabola.



[−6, 6] by [−4, 4]

$0 \le \theta \le 2\pi$

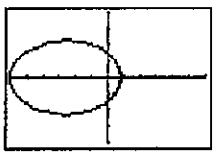
35. It is a hyperbola.



[−6, 6] by [−4, 4]

$0 \le \theta \le 2\pi$

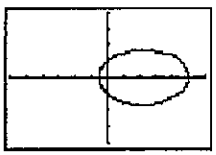
36. It is an ellipse.



[−6, 6] by [−4, 4]

$0 \le \theta \le 2\pi$

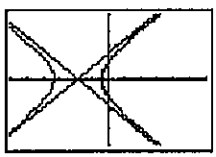
37. It is an ellipse.



[−6, 6] by [−4, 4]

$0 \le \theta \le 2\pi$

38. It is a hyperbola.



[−6, 6] by [−4, 4]

$0 \le \theta \le 2\pi$

39. $r = -1 + \sin\theta$

$$\frac{dy}{dx} = \frac{\dfrac{d}{d\theta}(-1+\sin\theta)\sin\theta}{\dfrac{d}{d\theta}(-1+\sin\theta)\cos\theta}$$

$$\frac{dy}{dx} = \frac{(2\sin\theta - 1)\cos\theta}{\cos^2\theta - \sin^2\theta - \sin\theta}$$

At $\theta = 0$: -1

At $\theta = \pi$: 1