81
3
243
16
259

CH10 LESSON 2 HOMEWORK

1 (No Calculator). The position of a moving particle in the xy-plane is given by parametric equations $x(t) = 9\cos t$ and $y(t) = 4\sin t$ for $t \ge 0$.

A. Find the speed of the particle at $t = \pi/3$

B. Find the acceleration vector at $t = 3$.

$v(t) = \langle -9\sin t, 4\cos t \rangle$

$a(t) = \langle -9\cos t, -4\sin t \rangle$

A] $\text{speed} = |v(\pi/3)| = \sqrt{[-9\sin(\pi/3)]^2 + [4\cos(\pi/3)]^2}$

$= \sqrt{81\left(\frac{3}{4}\right) + 16\left(\frac{1}{4}\right)}$

$= \sqrt{\frac{243+16}{4}} = \boxed{\frac{\sqrt{259}}{2}}$

B] $\boxed{a(3) = \langle -9\cos(3), -4\sin(3) \rangle}$

2 (No Calculator). A particle moves in the xy-plane so that any time t, $t > 0$, its coordinates are $x = e^t \sin t$ and $y = e^t \cos t$. Find the velocity vector at $t = \pi$.

$\vec{r}(t) = \langle e^t \sin t, e^t \cos t \rangle$

$\vec{v}(t) = \vec{r}\,'(t) = \langle \underbrace{e^t \cos t + \sin t \cdot e^t}_{\text{product Rule}}, -e^t \sin t + e^t \cos t \rangle$

$\vec{v}(\pi) = \langle -e^\pi + 0, 0 - e^\pi \rangle = \boxed{\langle -e^\pi, -e^\pi \rangle}$

3 (No Calculator). The velocity vector of a particle moving in the xy-plane is given by $\vec{v} = \langle 2\sin t, 3\cos t \rangle$ for $t \ge 0$. At $t = 0$, the particle is at the point (1, 1). What is the position vector at $t = 2$?

$x(2) = x(0) + \int_0^2 2\sin t \, dt = 1 - \left[2\cos t\right]_0^2 = 1 - 2\cos(2) + 2$

$= 3 - 2\cos(2)$

$y(2) = 1 + \int_0^2 3\cos t \, dt = 1 + \left[3\sin t\right]_0^2 = 1 + 3\sin(2) - 0$

$\boxed{(3 - 2\cos 2, 1 + 3\sin 2)}$

4 (Calculator Required). The velocity vector of a particle moving in the xy-plane is given by $\vec{v} = \left\langle \sqrt{1+t^2}, \sin\left(e^t - 4\right) \right\rangle$ for $t \ge 0$. At $t = 0$, the particle is at the point (-3, 1). What is the position vector at $t = 2$?

$x(2) = x(0) + \int_0^2 \sqrt{1+t^2}\, dt = -3 + \int_0^2 \sqrt{1+t^2}\, dt \approx -0.042$

$y(2) = y(0) + \int_0^2 \sin(e^t - 4)\, dt = 1 + \int_0^2 \sin(e^t - 4)\, dt \approx 0.468$

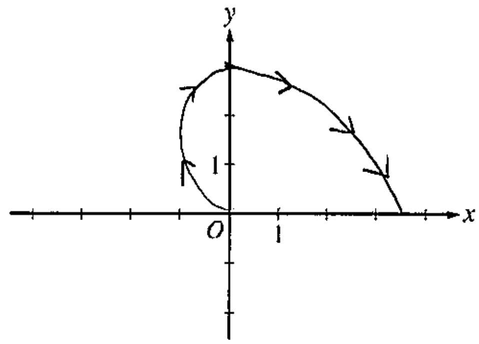
$\boxed{(-0.042, 0.468)}$

HW #5 (Calculator Required)

A particle moves in the xy-plane so that its position at any time t, $0 \le t \le \pi$, is given by

$x(t) = \frac{t^2}{2} - \ln(1 + t)$ and $y(t) = 3 \sin t$.

(a) Sketch the path of the particle in the xy-plane below. Indicate the direction of motion along the path.



(b) At what time t, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t), y(t))$ of the particle at this time?

(c) At what time t, $0 < t < \pi$, is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

(b) $x'(t) = t - \frac{1}{1+t} = 0$ at $t = 0.618$

$x'(t) < 0$ on $[0, 0.618]$

$x'(t) > 0$ on $(0.618, \pi]$

x is min at $t = 0.618$ b/c $x'(t)$ goes from negative to positive at $t=0.618$

$x(0.618) \approx -0.290$

$y(0.618) \approx 1.738$

$(-0.290, 1.738)$

(c) $x(t) = 0$ at $t = 1.286$

speed $= |\vec{v}'(1.286)| = \sqrt{\left(1.286 - \frac{1}{1+1.286}\right)^2 + (3\cos(1.286))^2} \approx 1.196$

$a(1.286) = \left\langle 1 + \frac{1}{(1+1.286)^2}, -3\sin(1.286)\right\rangle = \langle 1.191, -2.879\rangle$

HW #6 (No Calculator)

. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x-coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.

(a) Find $x(t)$ in terms of t.

(b) Find $\frac{dy}{dt}$ in terms of t.

(c) Find the location and speed of the particle at time $t = 4$.

a) $$x(t) = \int \left(\frac{dx}{dt}\right) dt = \int (2t+1)^{-1/2}\, dt = \frac{1}{2}\frac{2}{1}u^{1/2} + C = \sqrt{2t+1} + C$$

$u = 2t+1$

$du = 2\,dt$

$$x(0) = \sqrt{1} + C = -4 \qquad \therefore\ C = -5$$

$$x(t) = \sqrt{2t+1} - 5$$

b) $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{dy}{dt}}{(2t+1)^{-1/2}} = 3x^2 - 3 = 3\left(\sqrt{2t+1} - 5\right)^2 - 3$$

$$\therefore\ \frac{dy}{dt} = \frac{3\left(\sqrt{2t+1} - 5\right)^2 - 3}{\sqrt{2t+1}} \Bigg|_{t=4} = \frac{3(-2)^2 - 3}{3} = 3$$

OR: $$\frac{dy}{dt} = 3x^2\frac{dx}{dt} - 3\frac{dx}{dt} = 3\left(\sqrt{2t+1} - 5\right)^2 \frac{1}{\sqrt{2t+1}} - 3\left(\frac{1}{\sqrt{2t+1}}\right)$$

implicit

$(9 - 3)$

c) $$x(4) = \sqrt{9} - 5 = -2$$

$$y\,(x = -2) = (-2)^3 - 3(-2) = -8 + 6 = -2 \qquad \boxed{(-2, -2)}$$

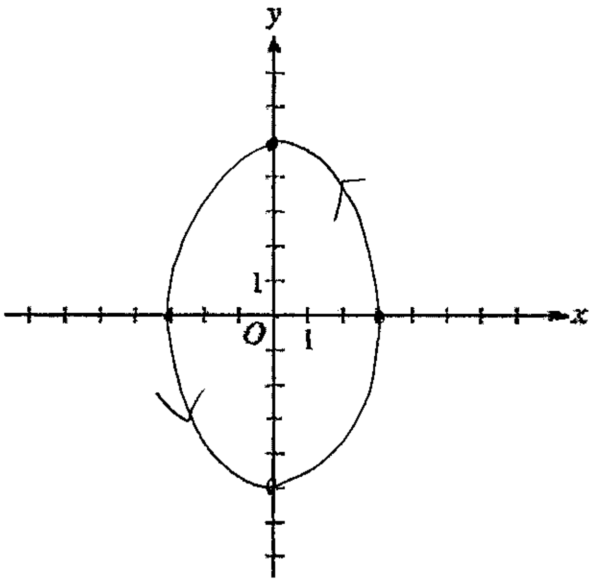
$$\text{Speed} = \sqrt{[x'(4)]^2 + [y'(4)]^2} = \sqrt{\left(\frac{1}{3}\right)^2 + (3)^2} = \boxed{\frac{\sqrt{82}}{3}}$$

HW #7 (Calculator Required)

During the time period from $t = 0$ to $t = 6$ seconds, a particle moves along the path given by $x(t) = 3\cos(\pi t)$ and $y(t) = 5\sin(\pi t)$.

(a) Find the position of the particle when $t = 2.5$. $x(2.5) = 0 \quad y(2.5) = 5 \quad (0, 5)$

(b) On the axes provided below, sketch the graph of the path of the particle from $t = 0$ to $t = 6$. Indicate the direction of the particle along its path.



$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = \cos^2(\pi t) + \sin^2(\pi t) = 1$$

$$\frac{x^2}{3} + \frac{y^2}{5} = 1 \quad \text{ellipse}$$

$x = 3\cos(\pi t) = 0$ graph $y = 3\cos(\pi x)$ and find zeros b/w $(0, 6)$

(c) How many times does the particle pass through the point found in part (a)? 3 times
$t = 0.5, 2.5, 4.5$

(d) Find the velocity vector for the particle at any time t.

(e) Write and evaluate an integral expression, in terms of sine and cosine, that gives the distance the particle travels from $t = 1.25$ to $t = 1.75$.

(d) $\vec{r}(t) = \langle 3\cos \pi t, 5\sin \pi t \rangle \quad \vec{v}(t) = \langle -3\pi \sin t, 5\pi \cos \pi t \rangle$

(e)
$$L = \int_{1.25}^{1.75} \sqrt{(-3\pi \sin t)^2 + (5\pi \cos \pi t)^2}\, dt \approx 5.392$$