[TEXTBOOK 10.1]

#9. $$\frac{dy}{dx} = \frac{\frac{1}{2}(3t)^{-1/2}\cdot 3}{-\frac{1}{2}(t+1)^{-1/2}} = -3\sqrt{\frac{t+1}{3t}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d\left(-3\sqrt{\frac{t+1}{3t}}\right)}{dt}}{-\frac{1}{2}(t+1)^{-1/2}} = \frac{-\frac{3}{2}\cancel{\left(\frac{t+1}{3t}\right)^{-1/2}}\cdot\left[\frac{3t-3(t+1)}{(3t)^2}\right]}{-\frac{1}{2}\cancel{(t+1)^{-1/2}}}$$

$$= 3\sqrt{3t}\left[\frac{\cancel{3t-3t}^{0}-3}{9t^2}\right] = \frac{-9\sqrt{3t}}{9t^2} = \frac{-\sqrt{3t}}{t^2} = \frac{-\sqrt{3}}{t^{3/2}} \checkmark$$

#11. $$\frac{dy}{dx} = \frac{3t^2}{2t-3}$$

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{3t^2}{2t-3}\right)}{2t-3} = \frac{\frac{(2t-3)(6t)-3t^2(2)}{(2t-3)^2}}{(2t-3)}$$

$$= \frac{(2t-3)(6t)-6t^2}{(2t-3)^3} = \frac{12t^2-18t-6t^2}{(2t-3)^3} = \frac{6t^2-18t}{(2t-3)^3} \checkmark$$

#16. $$\frac{dy}{dx} = \frac{\cancel{5}e^{5t}}{\frac{1}{5t}\cdot\cancel{5}} = 5t\left(e^{5t}\right)$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(5te^{5t}\right)}{\frac{1}{t}} = \frac{5t\left(5e^{5t}\right)+5e^{5t}}{\frac{1}{t}}$$

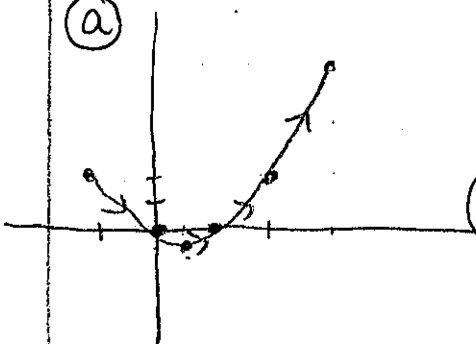
$$= 25t^2e^{5t}+5te^{5t}$$

$\frac{1}{4} - \frac{2}{4} \quad -\frac{1}{4}$

17.

t :	-2	-1	0	1	2	$-\frac{1}{2}$
x :	-1	0	1	2	3	$\frac{1}{2}$
y :	2	0	0	2	6	

a



$$\frac{dy}{dt} = 2t + 1 = 0 \qquad \frac{d^2y}{dt^2} = 2 > 0$$
$$t = -1/2$$

b $y(-1/2) = -\frac{1}{4}$ ← lowest (min y)

$x(-1/2) = \frac{1}{2}$ $\boxed{(1/2, -1/4)}$

#26 $\frac{dy}{dx} = \frac{3\cos t}{-3\sin t} = -\frac{\cos t}{\sin t}$

$\frac{dy}{dx} = 0 \quad -\cos t = 0 \quad \cos t = 0 \qquad t = \frac{\pi}{2} + 2\pi k \vee t = \frac{3\pi}{2} + 2\pi k$

$\therefore \quad x = -2 \quad y = 1 + 3 = 4 \qquad (-2, 4)$

$x = -2 \quad y = 1 - 3 = -2 \qquad (-2, -2)$

$\frac{dy}{dx}$ DNE $\quad \sin t = 0 \qquad t = 0 + 2\pi k$

$t = 2\pi + 2\pi k$

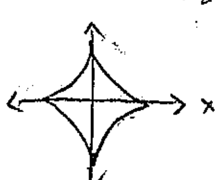
$x = -2 + 3 = 1 \quad y = 1 \qquad (1, 1)$

$x = -2 - 3 = -5 \quad y = 1 \qquad (-5, 1)$

#27 $L = \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2}\, dt = \int_0^{2\pi} dt = 2\pi$

#30 $L = 4\int_0^{\pi/2} \sqrt{(-6\cos^2 t \sin t)^2 + (6\sin^2 t \cos t)^2}\, dt = \cdots = 12$

NOT DIFF $\pi/2, \pi, 3\pi/2, 2\pi$ #43: $\boxed{21.010}$



[HW Answers 10.1 supplemental]

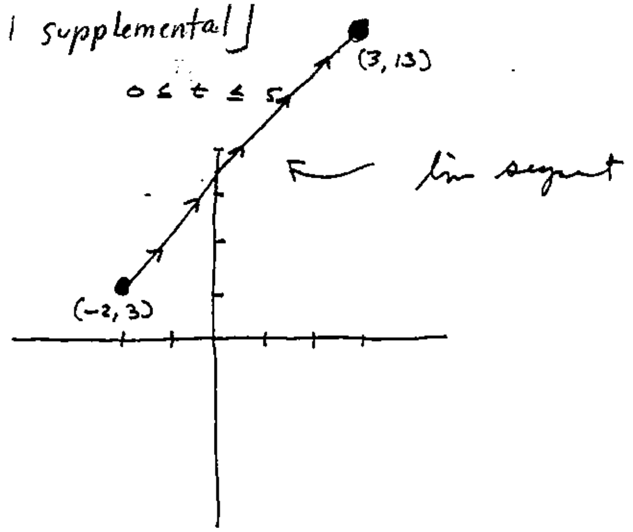
1. $x = t-2 \quad y = 2t+3 \qquad 0 \le t \le 5$

$t = x+2$

$-2 \le x \le 3 \qquad 3 \le y \le 13$

$y = 2(x+2)+3$

$y = 2x+7$



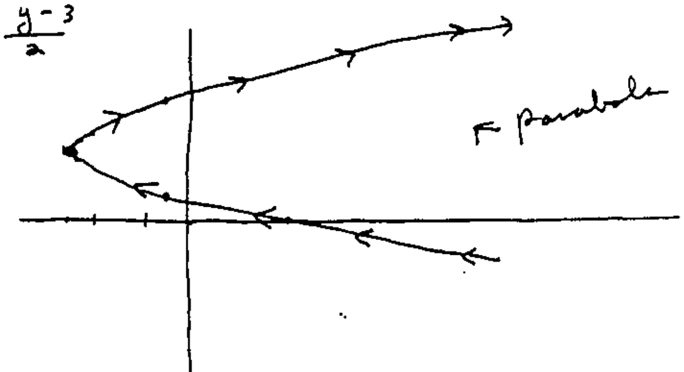
2. $x = 4t^2 - 5 \quad y = 2t+3 \qquad t \in \mathbb{R}$

$x \ge -5 \qquad y \in \mathbb{R}$

$t = \frac{y-3}{2}$

$x = 4\left(\frac{y-3}{2}\right)^2 - 5$

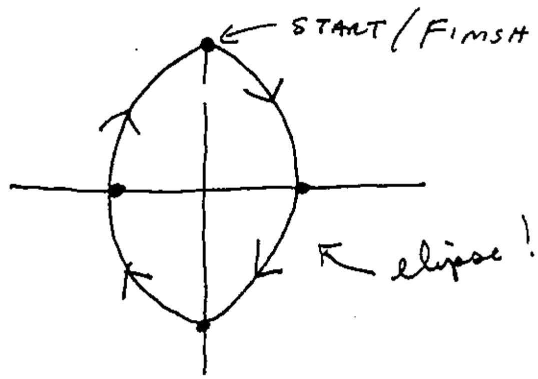
$x = y^2 - 6y + 4$



3. $x = 2\sin t \quad y = 3\cos t. \qquad 0 \le t \le 2\pi$

$\left(\frac{x}{2}\right)^2 = \sin^2 t \quad \left(\frac{y}{3}\right)^2 = \cos^2 t$

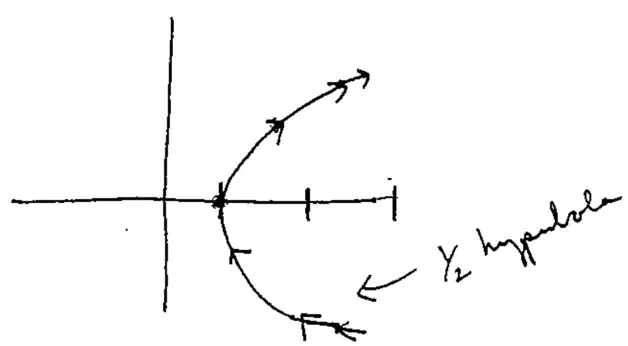
$\frac{x^2}{4} + \frac{y^2}{9} = 1$



4. $x = \sec t \quad y = \tan t \qquad -\pi/2 < t < \pi/2$

$\sec^2 t - \tan^2 t = 1$

$x^2 - y^2 = 1$



5. $x = t^2+1 \quad y = t^2-1 \quad -2 \le t \le 2$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{2t} = 1 \quad t \neq 0$$

$$M_T = 1$$
$$M_N = -1$$

6. $x = e^t \quad y = e^{-2t} \quad t \in \mathbb{R}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2e^{-2t}}{e^t} = -2e^{-3t}\Big|_{t=1} \quad M_T = -2e^{-3} \quad M_N = \frac{e^3}{2}$$

7. $x = -t^3 \quad y = -6t^2-18t \quad m = 2$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-12t-18}{-3t^2} = 2$$

$$4t+6 = 2t^2$$
$$2(t^2-2t-3) = 0$$
$$2(t-3)(t+1) = 0 \quad t = 3, -1$$

$(-27, -108)$

$(1, 12)$

8. $\frac{dy}{dx} = \frac{3t^2-12}{8t}$

horizontal

$3t^2-12=0$

$t = \pm 2 \quad (16, \pm 16)$

Vertical

$8t = 0$

$t = 0 \quad (0,0)$

$$\frac{d^2y}{dx^2} = \left[\frac{3t^2-12}{8t}\right]' \cdot \frac{1}{8t} = \boxed{\frac{24t^2+96}{(8t)^3}} \quad \text{OR} \quad \frac{3t^2+12}{8t^2}\cdot\frac{1}{8t} = \boxed{\frac{3t^2+12}{64t^3}}$$

9. $$L = \int_0^1 \sqrt{(10t)^2+(6t^2)^2}\,dt = \int_0^1 \sqrt{100t^2+36t^4}\,dt$$

$$= \int_0^1 2t\sqrt{25+9t^2}\,dt \cdots$$

use u-substitution

$u = 25+9t^2$

$du = 18t\,dt$

$dt = \frac{du}{18t}$

10. $$L = \int_0^{\pi/2} \sqrt{e^{2t}(\cos t - \sin t)^2 + e^{2t}(\cos t + \sin t)^2}\, dt$$

$$= \int_0^{\pi/2} \sqrt{e^{2t}(1+1)}\, dt$$

$$= \int_0^{\pi/2} e^t \sqrt{2} = \sqrt{2}\int_0^{\pi/2} e^t\, dt = \sqrt{2}e^t\Big]_0^{\pi/2} = \boxed{\sqrt{2}\left[e^{\pi/2} - 1\right]}$$

11. $$\int_0^4 2\pi(2t)\sqrt{(2t)^2 + (2)^2}\, dt \approx 578.83$$

12. $$\int_0^{\pi/2} 2\pi e^t \sin t\, e^t \sqrt{2} = 2\sqrt{2}\int_0^{\pi/2} e^{2t}\sin t\, dt$$

By Parts!

$$\int_0^{\pi/2} e^{2t}\sin t\, dt = -e^{2t}\cos t + \int 2e^{2t}\cos t$$

$u = e^{2t}$ $dv = \sin t$

$du = 2e^{2t}dt$ $v = -\cos t$

$u = 2e^{2t}$ $dv = \cos t$

$du = 4e^{2t}dt$ $v = \sin t$

$$= -e^{2t}\cos t + 2e^{2t}\sin t - 4\int e^{2t}\sin t$$

$$\int e^{2t}\sin t = -e^{2t}\cos t + 2e^{2t}\sin t - 4\int e^{2t}\sin t$$

$$5\int e^{2t}\sin t = -e^{2t}\cos t + 2e^{2t}\sin t$$

$$\int e^{2t}\sin t = \frac{-e^{2t}\cos t + 2e^{2t}\sin t}{5}\Bigg]_0^{\pi/2} = \frac{-e^{\pi}}{5} + \frac{1}{5}$$

$$2\sqrt{2}\int_0^{\pi/2} e^{2t}\sin t = 2\sqrt{2}\left[\frac{e^{-\pi}}{5} + \frac{1}{5}\right]$$