Lesson 2 Supp #1

a] $f(4) = 7 \qquad \dfrac{f'''(4)(x-4)^3}{3!} = -2(x-4)^3$

$\therefore f'''(4) = -2 \cdot 3! = -12$

b] $f(x) \approx P_3(x) = 7 - 3(x-4) + 5(x-4)^2 - 2(x-4)^3$

$f'(x) \approx -3 + 10(x-4) - 6(x-4)^2$

$f'(4.3) \approx -3 + 10(0.3) - 6(0.3)^2$

c]

$$g(x) \approx \int_4^x \left(7 - 3(t-4) + 5(t-4)^2 - 2(t-4)^3\right)dt$$

$$= \left[7t - \frac{3(t-4)^2}{2} + \frac{5(t-4)^3}{3} - \frac{2(t-4)^4}{4}\right]_4^x$$

$$= 7(x-4) - \frac{3(x-4)^2}{2} + \frac{5(x-4)^3}{3} - \frac{(x-4)^4}{2}$$

Lesson 2 Supp #2

a) $f(x) \approx 5 - 3x + \dfrac{x^2}{2} + \dfrac{4x^3}{3!} \qquad f(0.2) \approx 5 - 3(0.2) + \dfrac{(0.2)^2}{2} + \dfrac{4(0.2)^3}{3!}$

b) $g(x) = f(x^2) \approx 5 - 3(x^2) + \dfrac{(x^2)^2}{2}$

c) $h(x) = \displaystyle\int_0^x f(t)\,dt \approx \left[5t - \frac{3t^2}{2} + \frac{t^3}{2\cdot 3}\right]_0^x = 5x - \frac{3x^2}{2} + \frac{x^3}{6}$

PART (d)

$r(x) = \int r'(x)dx = \int f(x)dx = 5x - \dfrac{3x^2}{2} + \dfrac{x^3}{6} - \dots + C$

$r(0) = (0 + 0 + \dots) + C = 10 \therefore C = 10,$

$r(x) \approx 10 + 5x - \dfrac{3x^2}{2} + \dfrac{x^3}{6}$

Lesson 2 Supp #3 a) $f'(0) = \frac{1}{2} \qquad f^{17}(0) = \frac{17!}{18!} = \frac{1}{18}$

b) $g(x) = x f(x) = x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dots + \dfrac{x^{n+1}}{(n+1)!} + \dots$

c) $g(x) = e^x - 1 \qquad f(x) = \dfrac{g(x)}{x} = \begin{cases} \dfrac{e^x - 1}{x} ; & x \neq 0 \\ 1 ; & x = 0 \end{cases}$

d) $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{x^n}\right| = \lim_{n\to\infty}\frac{1}{n+2}|x| = 0 < 1$

The series converges for all real numbers.

Lesson 2 TEXTBOOK Problems

9.2 #4 $f(x) = e^{1-x} \approx P_5(x) = e - ex + \frac{ex^2}{2!} - \frac{ex^3}{3!} + \frac{ex^4}{4!} - \frac{ex^5}{5!}$

$f(0) = e^1$

$f'(x) = -e^{1-x}$ $f'(0) = -e^1$

$f''(x) = e^{1-x}$ $f''(0) = e^1$

$f'''(x) = -e^{1-x}$ $f'''(0) = -e^1$

$f(x) = \sum_0^{\infty} (-1)^n \frac{e}{n!} x^n$ for $-\infty < x < \infty$

9.2 #14 $f(x) = e^{x/2} = e^{1/2} + \frac{1}{2}e^{1/2}(x-1) + \frac{\frac{1}{4}e^{1/2}(x-1)^2}{2!} + \cdots + \sum_0^{\infty} \frac{e^{1/2}(x-1)^n}{2^n n!}$

$f(1) = e^{1/2}$

$f'(1) = \frac{1}{2}e^{1/2}$

$f''(1) = \frac{1}{4}e^{1/2}$

$f'''(1) = \frac{1}{8}e^{1/2}$

for $-\infty < x < \infty$

9.2 #19. $f(x) = \sin x \approx P_3(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x - \pi/4) - \frac{\sqrt{2}}{4}(x - \pi/2)^2 - \frac{\sqrt{2}}{12}(x - \frac{\pi}{4})^3$

$f(\pi/4) = \sin(\pi/4) = \frac{\sqrt{2}}{2}$

$f'(\pi/4) = \cos(\pi/4) = \frac{\sqrt{2}}{2}$

$f''(\pi/4) = -\sin(\pi/4) = -\frac{\sqrt{2}}{2}$

$f'''(\pi/4) = -\cos(\pi/4) = -\frac{\sqrt{2}}{2}$

for $-\infty < x < \infty$

9.2 #23. $f(x) \approx P_3(x) = 4 - (x-1) + 3\frac{(x-1)^2}{2} + \frac{2(x-1)^3}{3!}$

$f(1.2) \approx 4 - (0.2) + \frac{3(0.2)^2}{2} + \frac{(0.2)^3}{3}$

$f'(x) \approx Q_2(x) = -1 + 3(x-1) + (x-1)^2$

$f'(1.2) \approx -1 + 3(0.2) + (0.2)^2$

9.2 #33. $\ln(x) = \ldots + \frac{2}{(2)^3} \cdot \frac{(x-3)^3}{3!} + \ldots$

$f'(x) = \frac{1}{x}$

$f''(x) = \frac{-1}{x^2}$

$f'''(x) = \frac{2}{x^3}$

$f^{iv}(x) = \frac{-6}{x^4}$

$f'''(2) = \frac{2}{(2)^3}$

Coefficient $\frac{2}{8 \cdot 6} = \frac{1}{8 \cdot 3} = \boxed{\frac{1}{24}}$