CH 9A – Lesson 2 HW Solutions : ONLY ESSENTIAL PARTS ARE REPORTED
You must follow formal format
like in notes

1) 9.4 Pg. 511 #35 : $\sum_0^\infty n^2 e^{-n}$ ♡ RATIO TEST

ii) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^2}{e^{n+1}} \cdot \frac{e^n}{n^2} = \frac{1}{e} \lim_{n\to\infty} \frac{(n+1)^2}{n^2} = \frac{1}{e} < 1 \therefore \sum_0^\infty n^2 e^{-n}$ converges

2) 9.4 pg 511 # 37 ; $\sum_1^\infty \frac{(n+3)!}{3!\,n!\,3^n}$ ♡ RATIO TEST

ii) $\lim_{n\to\infty} \frac{(n+4)!}{3!\,(n+1)!\,3^{n+1}} \cdot \frac{3!\,n!\,3^n}{(n+3)!} = \lim_{n\to\infty} \frac{n+4}{n+1} \cdot \frac{1}{3} = \frac{1}{3} < 1 \therefore \sum_1^\infty \frac{(n+1)!}{3!\,n!\,3^n}$ Converges

3) 9.4 pg 511 # 40 : $\sum_1^\infty n!\,e^{-n}$ ♡ RATIO TEST

ii) $\lim_{n\to\infty} = \frac{(n+1)!}{e^{n+1}} \cdot \frac{e^n}{n!} = \lim_{n\to\infty} \frac{1}{e}(n+1) \to \infty > 1 \therefore \sum_1^\infty n!\,e^{-n}$ diverges

4) 9.4 pg 511 # 43 : $\sum_1^\infty \frac{n!}{(2n+1)!}$ ♡ RATIO TEST

ii) $\lim_{n\to\infty} \frac{(n+1)!}{(2n+2+1)!} \cdot \frac{(2n+1)!}{n!} = \lim_{n\to\infty} \frac{(n+1)}{(2n+3)(2n+2)} = 0 < 1 \therefore \sum \frac{n!}{(2n+1)!}$ converges

5) 9.4 pg 511 # 44 : $\sum_1^\infty \frac{n!}{n^n}$ ♡ RATIO TEST

ii) $\lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \lim_{n\to\infty} \frac{(n+1)\,n^n}{(n+1)^n(n+1)} = \lim_{n\to\infty} \frac{n^n}{(n+1)^n} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n$

$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n} = \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^n\right]^{-1} = e^{-1} = \frac{1}{e} < 1 \therefore \sum_1^\infty \frac{n!}{n^n}$ converges

6) 9.4 pg 511 # 23 : $\sum_0^\infty \frac{(x-1)^{2n}}{4^n} = \sum_0^\infty \left(\frac{(x-1)^2}{4}\right)^n$ Geom : $a_1 = 1 \quad r = \frac{(x-1)^2}{4}$

$\frac{(x-1)^2}{4} < 1 \to |(x-1)| < 2 \to -2 < x-1 < 2 \to -1 < x < 3$

$S = \frac{-4}{x^2-2x-3}$ on $-1 < x < 3$

7) 9.4 pg 511 # 26 : $\sum_0^\infty (\ln x)^n \quad |\ln x| < 1 \to -1 < \ln x < 1$

for $e^{-1} < x < e \to S = \frac{1}{1-\ln x}$ Geom : $a_1 = 1 \quad r = \ln x$

8) 9.4 Pg. 511 # 27 $\sum_{0}^{\infty}\left(\frac{x^2-1}{3}\right)^n$ Geom: $a_1=1$ $r=\frac{x^2-1}{3}$

$\left|\frac{x^2-1}{3}\right|<1 \rightarrow |x^2-1|<3 \xrightarrow{*} -2<x<2$ see below

I.C: $-2<x<2$ $S=\frac{3}{4-x^2}$ for $-2<x<2$

9) 9.4 Pg 511 # 28 $\sum_{0}^{\infty}\left(\frac{\sin x}{2}\right)^n$ Geom: $a_1=1$ $r=\frac{\sin x}{2}$

$\left|\frac{\sin x}{2}\right|<1 \rightarrow |\sin x|<2$ True for all x

$S=\frac{1}{1-\frac{\sin x}{2}}=\frac{2}{2-\sin x}$ $x\in\mathbb{R}$ interval of convergence: $-\infty<x<\infty$

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* $|x^2-1|<3$

$-3<x^2-1<3$

$-1\leq x^2-1<3$ ∵ b/c $x^2\geq 0$

$0\leq x^2<4$

$x^2<4$ $|x|<2$

$\boxed{-2<x<2}$