

L'HOPITAL'S RULE PRACTICE SET #2

$$\textcircled{1} \lim_{x \rightarrow 0} \frac{\sin x}{2x} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{\cos x}{2} = \boxed{\frac{1}{2}}$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{5x}{\tan x} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{5}{\sec^2 x} = \frac{5}{1} = \boxed{5}$$

$$\textcircled{3} \lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{x^2 - 25} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 5} \frac{\frac{1}{2}(x-1)^{-1/2}}{2x} = \frac{\frac{1}{2}}{2 \cdot 5} = \boxed{\frac{1}{20}}$$

$$\textcircled{4} \lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{2x^2 + 3x - 9} \quad \begin{matrix} 9 - 6 - 3 \\ 18 - 9 - 9 \end{matrix} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow -3} \frac{2x + 2}{4x + 3} = \frac{-4}{-9} = \boxed{\frac{4}{9}}$$

$$\textcircled{5} \lim_{x \rightarrow 0} \frac{\sin x - x}{\tan x - x} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sec^2 x - 1} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{-\sin x}{2 \sec x \operatorname{sech} x}$$

$$= \lim_{x \rightarrow 0} \frac{-\sin x}{\frac{2}{\cos^2 x} \cdot \frac{\sin x}{\cos x}}$$

$$= \lim_{x \rightarrow 0} -\frac{\cos^3 x}{2} = \boxed{-\frac{1}{2}}$$

$$\textcircled{6} \lim_{x \rightarrow 0} \frac{x+1-e^x}{x^2} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{1-e^x}{2x} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{-e^x}{2} = \boxed{-\frac{1}{2}}$$

$$\textcircled{7} \lim_{x \rightarrow 0} \frac{x-\sin x}{x^3} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{1-\cos x}{3x^2} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{+\sin x}{6x} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{\cos x}{6} = \boxed{\frac{1}{6}}$$

$$\textcircled{8} \lim_{x \rightarrow \pi/2} \frac{1-\sin x}{\cos x} \quad \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$= \lim_{x \rightarrow \pi/2} \frac{-\cos x}{-\sin x} = \frac{0}{1} = \boxed{0}$$

$$\textcircled{9} \lim_{x \rightarrow \pi/2} \frac{1+\sin x}{\cos^2 x} = \boxed{\infty} \quad \left(\frac{2}{0} \right)$$

$$\textcircled{10} \lim_{x \rightarrow 0^+} \frac{\ln x}{\cot x} \quad -\left\{ \begin{matrix} \infty \\ \infty \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\csc^2 x} \quad -\left\{ \begin{matrix} \infty \\ \infty \end{matrix} \right\}$$

$$= \lim_{x \rightarrow 0^+} \frac{-\sin^2 x}{x}$$

$$= \lim_{x \rightarrow 0^+} \frac{-2 \sin x \cos x}{1} = \boxed{0}$$

$$\textcircled{11} \lim_{x \rightarrow 0} \frac{x \cos x + e^{-x}}{x^2} = \infty$$

$\frac{1}{0}$

$$(12) \lim_{x \rightarrow \infty} \frac{2x^2 + 3x + 1}{5x^2 + x + 4} = \boxed{\frac{2}{5}} \leftarrow \text{Precalc}$$

$$(13) \lim_{x \rightarrow 0} \frac{\sin^{-1}(2x)}{\sin^{-1}(x)} \quad \left\{ \begin{array}{l} 0 \\ 0 \end{array} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-(2x)^2}} \cdot 2}{\frac{1}{\sqrt{1-x^2}}}$$

$$= \lim_{x \rightarrow 0} \frac{2\sqrt{1-x^2}}{\sqrt{1-4x^2}} = \frac{2}{1} = \boxed{2}$$

$$(14) \lim_{x \rightarrow -\infty} \frac{3-3^x}{5-5^x} = \boxed{\frac{3}{5}} \leftarrow \text{Precalc}$$

$$(15) \lim_{x \rightarrow 1} \frac{2x^3 - 5x^2 + 6x - 3}{x^3 - 2x^2 + x - 1} = \frac{0}{-1} = \boxed{0}$$

$$\frac{2-5+6-3}{1-2+1-1}$$

$$(16) \lim_{x \rightarrow 0} \frac{x - \tan^{-1}x}{x \sin x} \quad \left\{ \begin{array}{l} 0 \\ 0 \end{array} \right\}$$

$$= \lim_{x \rightarrow 0} \frac{1 - \frac{1}{1+x^2}}{x \cos x + \sin x} \quad \left\{ \begin{array}{l} 0 \\ 0 \end{array} \right\} \quad -(1+x^2)^{-1}$$

$$= \lim_{x \rightarrow 0} \frac{(1+x^2)^{-2} \cdot 2x}{-x \sin x + \cos x + \cos x} = \frac{0}{2} = \boxed{0}$$

Product Rule