L'HOPITAL'S RULE PRACTICE SET #2

① $\lim_{x\to 0} \frac{\sin x}{2x}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{\cos x}{2} = \boxed{\frac{1}{2}}$

② $\lim_{x\to 0} \frac{5x}{\tan x}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{5}{\sec^2 x} = \frac{5}{1} = \boxed{5}$

③ $\lim_{x\to 5} \frac{\sqrt{x-1} - 2}{x^2 - 25}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 5} \frac{\frac{1}{2}(x-1)^{-1/2}}{2x} = \frac{\frac{1}{2\sqrt{4}}}{10} = \boxed{\frac{1}{40}}$

④ $\lim_{x\to -3} \frac{x^2+2x-3}{2x^2+3x-9}$ $9-6-3$, $18-9-9$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to -3} \frac{2x+2}{4x+3} = \frac{-4}{-9} = \boxed{\frac{4}{9}}$

⑤ $\lim_{x\to 0} \frac{\sin x - x}{\tan x - x}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{\cos x - 1}{\sec^2 x - 1}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{-\sin x}{2\sec x \sec x \tan x}$

$= \lim_{x\to 0} \frac{-\sin x}{\frac{2}{\cos^2 x} \cdot \frac{\sin x}{\cos x}}$

$= \lim_{x\to 0} -\frac{\cos^3 x}{2} = \boxed{-\frac{1}{2}}$

⑥ $\lim_{x\to 0} \frac{x+1-e^x}{x^2}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{1-e^x}{2x}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{-e^x}{2} = \boxed{\frac{-1}{2}}$

⑦ $\lim_{x\to 0} \frac{x-\sin x}{x^3}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{1-\cos x}{3x^2}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{+\sin x}{6x}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \frac{\cos x}{6} = \boxed{\frac{1}{6}}$

⑧ $\lim_{x\to \pi/2} \frac{1-\sin x}{\cos x}$ $\left\{\frac{\div}{\div}\right\}$

$= \lim_{x\to \pi/2} \frac{-\cos x}{-\sin x} = \frac{0}{1} = \boxed{0}$

⑨ $\lim_{x\to \pi/2} \frac{1+\sin x}{\cos^2 x} = \boxed{\infty}$ $\frac{2}{0}$

⑩ $\lim_{x\to 0^+} \frac{\ln x}{\cot x}$ $-\left\{\frac{\infty}{\infty}\right\}$

$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc^2 x}$ $-\left\{\frac{\infty}{\infty}\right\}$

$= \lim_{x\to 0^+} \frac{-\sin^2 x}{x}$

$= \lim_{x\to 0^+} \frac{-2\sin x \cos x}{1} = \boxed{0}$

⑪ $\lim_{x\to 0} \frac{x\cos x + e^{-x}}{x^2} = \infty$ $\frac{1}{0}$

(12) $\lim_{x\to\infty} \dfrac{2x^2+3x+1}{5x^2+x+4} = \boxed{\dfrac{2}{5}}$ ← Precalc

(13) $\lim_{x\to 0} \dfrac{\sin^{-1}(2x)}{\sin^{-1}(x)}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \dfrac{\frac{1}{\sqrt{1-(2x)^2}}\cdot 2}{\frac{1}{\sqrt{1-x^2}}}$

$= \lim_{x\to 0} \dfrac{2\sqrt{1-x^2}}{\sqrt{1-4x^2}} = \dfrac{2}{1} = \boxed{2}$

(14) $\lim_{x\to -\infty} \dfrac{3-3^{x}}{5-5^{x}} = \boxed{\dfrac{3}{5}}$ ← PreCalc

(15) $\lim_{x\to 1} \dfrac{2x^3-5x^2+6x-3}{x^3-2x^2+x-1} = \dfrac{0}{-1} = \boxed{0}$

$\dfrac{2-5+6-3}{1-2+1-1}$

(16) $\lim_{x\to 0} \dfrac{x-\tan^{-1}x}{x\sin x}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{x\to 0} \dfrac{1-\frac{1}{1+x^2}}{x\cos x+\sin x}$ $\left\{\frac{0}{0}\right\}$

$-(1+x^2)^{-1}$

$= \lim_{x\to 0} \dfrac{(1+x^2)^{-2}\cdot 2x}{-x\sin x+\cos x+\cos x} = \dfrac{0}{2} = \boxed{0}$

Product Rule