CH8 Lesson 1 L'Hopital's Rule (8.2)
HW Solutions

#5 $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2} \left\{\dfrac{0}{0}\right\} = \lim\limits_{x\to 0} \dfrac{\sin x}{2x} \left\{\dfrac{0}{0}\right\} = \lim\limits_{x\to 0} \dfrac{\cos x}{2} = \boxed{\dfrac{1}{2}}$

#13 $\lim\limits_{x\to \pi} \dfrac{\csc x}{1+\cot x} \left\{\dfrac{-\infty}{\infty}\right\} = \lim\limits_{x\to \pi} \dfrac{-\csc x \cot x}{-\csc^2 x} = \lim\limits_{x\to\pi} \dfrac{\cot x}{\csc x}$

$= \lim\limits_{x\to\pi} \cos x = \boxed{-1}$

#17 $\lim\limits_{x\to 0^+} (x\cdot \ln x) \;\{-0\cdot\infty\} = \lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}} \left\{\dfrac{\infty}{\infty}\right\}$

$= \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim\limits_{x\to 0^+} -x = \boxed{0}$

#21 $\lim\limits_{x\to 0} (e^x + x)^{1/x} \;\{1^\infty\}$ let $y = (e^x+x)^{1/x}$

$\ln y = \frac{1}{x}\ln(e^x+x)$

BUT FIRST: $\lim\limits_{x\to 0} \ln y = \lim\limits_{x\to 0} \frac{1}{x}\ln(e^x+x) \left\{\dfrac{0}{0}\right\} = \lim\limits_{x\to 0} \dfrac{\frac{1}{e^x+x}(e^x+1)}{1}$

$= \dfrac{(e^0+1)}{(e^0+0)} = \dfrac{2}{1} = 2$

AND NOW: $\lim\limits_{x\to 0} (e^x+x)^{1/x} = \lim\limits_{x\to 0} y = \lim\limits_{x\to 0} e^{\ln y} = e^{\lim\limits_{x\to 0}\ln y} = \boxed{e^2}$

#25 $\lim\limits_{x\to 0^+} \left(1+\frac{1}{x}\right)^x \;\{1^0\}$ let $y = \left(1+\frac{1}{x}\right)^x$

$\ln y = x\ln\left(1+\frac{1}{x}\right)$

BUT FIRST: $\lim\limits_{x\to 0^+} \ln y = \lim\limits_{x\to 0^+} x\ln\left(1+\frac{1}{x}\right) \;\{0\cdot\infty\} = \lim\limits_{x\to 0^+} \dfrac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \left\{\dfrac{\infty}{\infty}\right\}$

$= \lim\limits_{x\to 0^+} \dfrac{\frac{1}{\left(1+\frac{1}{x}\right)}\cdot\left(-\frac{1}{x^2}\right)}{\frac{-1}{x^2}} = \lim\limits_{x\to 0^+} \dfrac{1}{1+\frac{1}{x}} = \lim\limits_{x\to 0^+} \dfrac{1}{\frac{x+1}{x}}$

simplify

$= \lim\limits_{x\to 0^+} \dfrac{x}{x+1} = 0$

AND NOW: $\lim\limits_{x\to 0^+} \left(1+\frac{1}{x}\right)^x = \lim\limits_{x\to 0^+} y = \lim\limits_{x\to 0^+} e^{\ln y} = e^{\lim\limits_{x\to 0^+}\ln y} = e^0 = \boxed{1}$

#31 $\lim_{x\to\infty}(1+x)^{1/x}$ $\{\infty^0\}$ let $y=(1+x)^{1/x}$

$\ln y = \frac{1}{x}\ln(1+x)$

BUT FIRST: $\lim_{x\to\infty}\ln y = \lim_{x\to\infty}\frac{1}{x}\ln(1+x)$ $\left\{\frac{\infty}{\infty}\right\} = \lim_{x\to\infty}\frac{\frac{1}{1+x}\cdot(1)}{1} = 0$

AND NOW $\lim_{x\to\infty}(1+x)^{1/x} = \lim_{x\to\infty} y = \lim_{x\to\infty} e^{\ln y} = e^{\lim_{x\to\infty}\ln y} = e^0 = \boxed{1}$

#33 $\lim_{\theta\to 0}\frac{\sin\theta^2}{\theta}\left\{\frac{0}{0}\right\} = \lim_{\theta\to 0}\frac{\cos(\theta^2)\cdot 2\theta}{1} = \boxed{0}$

#35 $\lim_{x\to\infty}\frac{\log_2 x}{\log_3(x+3)} = \lim_{x\to\infty}\left(\frac{\ln 3}{\ln 2}\right)\frac{\ln x}{\ln(x+3)}$ $\left\{\frac{\infty}{\infty}\right\} = \frac{\ln 3}{\ln 2}\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{x+3}}$

$= \frac{\ln 3}{\ln 2}\lim_{x\to\infty}\frac{x+3}{x}$ $\left\{\frac{\infty}{\infty}\right\} = \frac{\ln 3}{\ln 2}\lim_{x\to\infty}\frac{1}{1} = \boxed{\frac{\ln 3}{\ln 2}}$

#37 $\lim_{y\to\pi/2}\left(\frac{\pi}{2}-y\right)\tan y$ $\{0\cdot\infty\} = \lim_{y\to\pi/2}\frac{\pi/2-y}{\cot y}$ $\left\{\frac{0}{0}\right\}$

$= \lim_{y\to\pi/2}\frac{-1}{-\csc^2 y} = \lim_{y\to\pi/2}\sin^2 y = \boxed{1}$

#39 $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sqrt{x}}\right) = \lim_{x\to 0^+}\left(\frac{1}{x}-\frac{\sqrt{x}}{x}\right) = \lim_{x\to 0^+}\left(\frac{1-\sqrt{x}}{x}\right) \to \boxed{\infty}$

#41 Review of Precalc $\boxed{0}$

#43 $\lim_{x\to\infty}(1+2x)^{1/2\ln x}$ $\{\infty^0\}$ let $y=(1+2x)^{1/2\ln x}$

$\ln y = \frac{1}{2\ln x}\ln(1+2x)$

BUT FIRST $\lim_{x\to\infty}\ln y = \lim_{x\to\infty}\frac{\ln(1+2x)}{2\ln(x)}$ $\left\{\frac{\infty}{\infty}\right\} = \lim_{x\to\infty}\frac{\frac{1}{1+2x}\cdot\cancel{2}}{\frac{\cancel{2}}{x}} = \lim_{x\to\infty}\frac{x}{1+2x}$

$= \lim_{x\to\infty}\frac{1}{2} = \frac{1}{2}$

AND NOW $\lim_{x\to a}(1+2x)^{1/2\ln x} = \cdots = \boxed{e^{1/2}}$

#45 In class $\boxed{e}$

#47 $\lim\limits_{x\to 1^+} x^{1/(1-x)}$ $\{1^\infty\}$ let $y = x^{1/(1-x)}$

$\ln y = \frac{\ln x}{1-x}$

But first $\lim\limits_{x\to 1^+} \ln y = \lim\limits_{x\to 1^+} \frac{\ln x}{1-x} \left\{\frac{0}{0}\right\} = \lim\limits_{x\to 1^+} \frac{\frac{1}{x}}{-1} = \lim\limits_{x\to 1^+} -\frac{1}{x} = -1$

And now $\lim\limits_{x\to 1^+} x^{1/(1-x)} = \dots = \boxed{e^{-1}}$

#49 $\lim\limits_{x\to 1} \frac{x^3-1}{4x^3-x-3} \left\{\frac{0}{0}\right\} = \lim\limits_{x\to 1} \frac{3x^2}{12x^2-1} = \boxed{\frac{3}{11}}$

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$$\lim_{x\to 1} \frac{\int_1^x \cos t\,dt}{x^2-1} \left\{\frac{0}{0}\right\} = \lim_{x\to 1} \frac{\cos x}{2x} = \boxed{\frac{\cos(1)}{2}}$$

62: $\boxed{\text{FALSE}}$ if $g'(a) = 0$

TRUE if $g'(a) \neq 0$ $\lim\limits_{x\to a} \frac{f(x)}{g(x)} \left\{\frac{0}{0}\right\} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)} = \frac{f'(a)}{g'(a)}$ ✓

63: $\boxed{\text{False}}$ Answer $\boxed{1}$

64: C

65: D

66: B

67: E