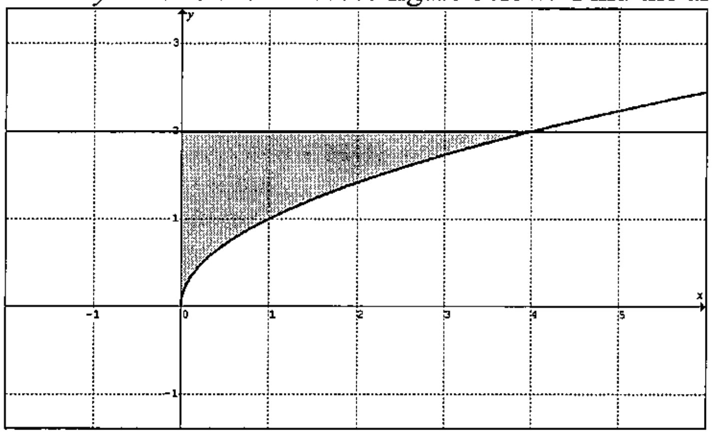
BC:Q302 LESSON 1 HOMEWORK

1. NO CALCULATOR. Let R be the shaded region enclosed by the graphs of $y=\sqrt{x}$, $y=2$, and the y-axis as shown in the figure below. Find the area of region R.



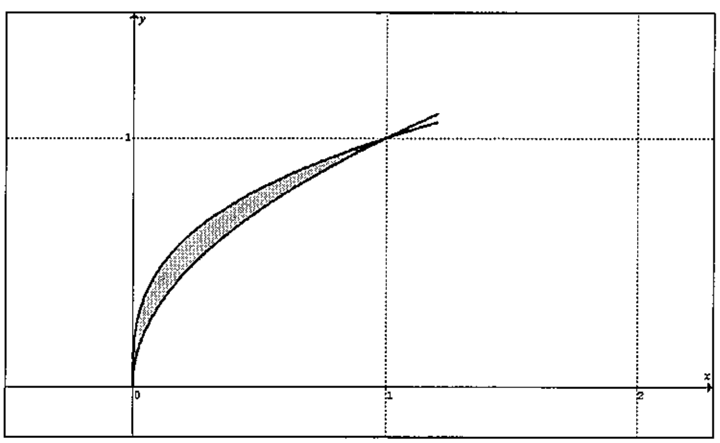
$$A=\int_0^4 \left[2-\sqrt{x}\right]dx$$

$$=\left[2x-\frac{2x^{3/2}}{3}\right]_0^4$$

$$=8-\frac{16}{3}=\boxed{\frac{8}{3}}$$

ALT: $\int_{y=0}^{y=2}\left[y^2-0\right]dy=\left[\frac{y^3}{3}\right]_0^2=\frac{8}{3}$ ✓

2. NO CALCULATOR. Let R be the shaded region enclosed by the graphs of $x=y^3$, $x=y^2$, and the x-axis as shown in the figure below. Find the area of region R.

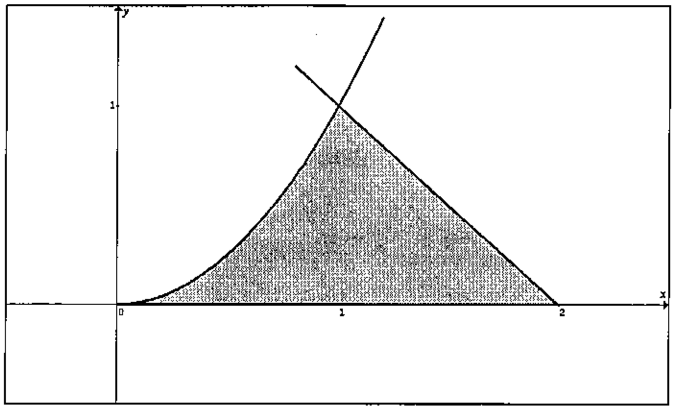


$$A=\int_{y=0}^{y=1}\left[y^2-y^3\right]dy=\left[\frac{y^3}{3}-\frac{y^4}{4}\right]_0^1$$

$$=\frac{1}{3}-\frac{1}{4}=\boxed{\frac{1}{12}}$$

ALT: $\int_{x=0}^{x=1}\left[x^{1/3}-x^{1/2}\right]dx=\left[\frac{3x^{4/3}}{4}-\frac{2x^{3/2}}{3}\right]_0^1=\frac{3}{4}-\frac{2}{3}=\frac{1}{12}$ ✓

3. NO CALCULATOR. Let R be the shaded region enclosed by the graphs of $y = x^2$, $x + y = 2$, and the x-axis as shown in the figure below. Find the area of region R.



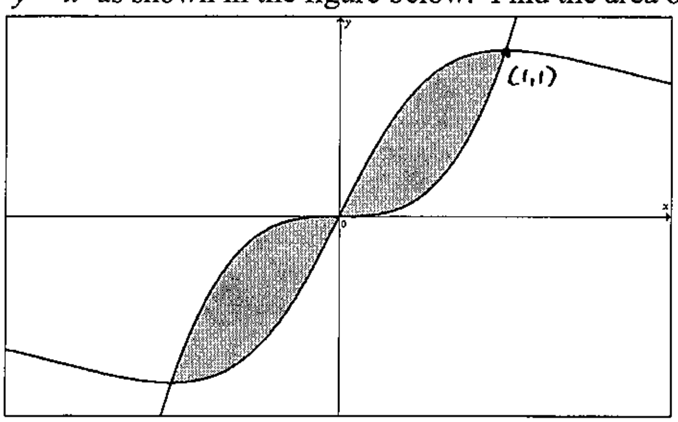
$$A = \int_{y=0}^{y=1} \left[(2-y) - y^{1/2}\right] dy$$

$$= \left[2y - \frac{y^2}{2} - \frac{2y^{3/2}}{3}\right]_0^1$$

$$= 2 - \frac{1}{2} - \frac{2}{3} = \boxed{\frac{5}{6}}$$

ALT: $\displaystyle\int_{x=0}^{x=1} [x^2]\, dx + \int_1^2 [2-x]\, dx = \left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \left[(4-2) - \left(2 - \frac{1}{2}\right)\right] = \frac{5}{6}$

4. NO CALCULATOR. Let R be the shaded region enclosed by the graphs of $y = \dfrac{2x}{x^2+1}$ and $y = x^3$ as shown in the figure below. Find the area of region R.



Note: the graphs intersect at (1,1)

$$A = 2\int_0^1 \left[\frac{2x}{x^2+1} - x^3\right] dx$$

$$= 2\left[\ln(x^2+1) - \frac{x^4}{4}\right]_0^1$$

$$= 2\left[\left(\ln(2) - \frac{1}{4}\right) - \left(\cancel{\ln(1)}^{\,0} - 0\right)\right]$$

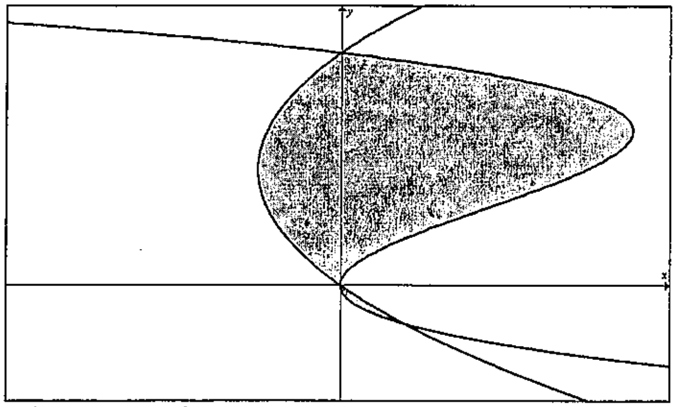
$$= \boxed{2\ln(2) - \frac{1}{2}}$$

$\displaystyle\int \frac{2x}{x^2+1}\, dx$

$u = x^2 + 1$

$du = 2x\, dx$

5. CALCULATOR PERMITTED. Let R be the shaded region enclosed by the graphs of $x = 2y^2 - 2y$ and $x = 12y^2 - 12y^3$ as shown in the figure below. Find the area of region R.



$f(y) = 2y^2 - 2y \qquad g(y) = 12y^2 - 12y^3$

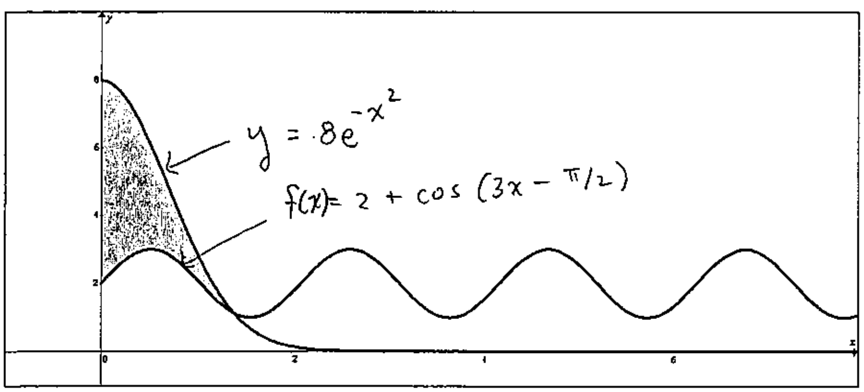
$f(y) = g(y)$ at $y = 1, 0, -\frac{1}{6}$

(If you are up for an extreme arithmetic challenge, try to find the answer to #5 without a calculator)

$$A = \int_{-1/6}^{0} \left[(2y^2 - 2y) - (12y^2 - 12y^3)\right] dy + \int_{0}^{1} \left[(12y^2 - 12y^3) - (2y^2 - 2y)\right] dy = \boxed{\frac{1741}{1296}}$$

$$A = \int_{-1/6}^{0} [f(y) - g(y)]\, dy + \int_{0}^{1} [g(y) - f(y)]\, dy = \frac{1741}{1296}$$

6. CALCULATOR REQUIRED Let R be the shaded region in the first quadrant enclosed by the graphs of $x = \sqrt{-\ln(y/8)}$ and $f(x) = 2 + \cos(3x - \pi/2)$ as shown in the figure below. Find the area of region R. Hint: You must perform preliminary work before using your calculator.



$y = f(x)$ at $x = 1.399$

$$x = \sqrt{-\ln(y/8)}$$

$$x^2 = -\ln(y/8)$$

$$-x^2 = \ln(y/8)$$

$$e^{-x^2} = \frac{y}{8}$$

$$8e^{-x^2} = y$$

$$A = \int_{0}^{1.399} \left[\left(8e^{-x^2}\right) - \left(2 + \cos(3x - \pi/2)\right)\right] dx = \boxed{3.455}$$

#(1e) $$V = \pi \int_0^2 (y^2)^2 \, dy = \pi \int_0^2 y^4 \, dy$$

#(3b) $$V = \pi \int_0^1 (x^2)^2 \, dx + \pi \int_1^2 (2-x)^2 \, dx$$